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array_shift_wf  THM

⊢  ∀T:Type, i:ℤ, j:{i...}, a:[T]Array{i..j⁻}, m:ℤ.
|  a[++m] ∈ [T]Array{i+m..j+m⁻}  by UnivCD
|
1. T : Type
2. i : ℤ
3. j : {i...}
4. a : [T]Array{i..j⁻}
5. m : ℤ
⊢  a[++m] ∈ [T]Array{i+m..j+m⁻}  by Analyze 4 THEN AbSetMemEqTypeCD
|
|\
| 4. a : [T]Array
| 5. a.l = i & a.u = j
| 6. m : ℤ
| ⊢  a[++m] ∈ [T]Array
| | by Unfolds [‘array‘;‘array_shift‘] 0 THEN Analyze 4
| |    THEN
| |    Analyze 5
| |
| 4. n : ℤ
| 5. m1 : {n...}
| 6. a2 : {n..m1⁻}→T
| 7. <n,m1,a2>.l = i & <n,m1,a2>.u = j
| 8. m : ℤ
| ⊢  <(<n,m1,a2>.l+m),(<n,m1,a2>.u+m),(λj.<n,m1,a2>[(j-m)])>
| |  ∈ n:ℤ×m:{n...}×{n..m⁻}→T
| | by Unfolds [‘array_el‘;‘array_lb‘;‘array_ub‘] 0
| |    THEN
| |    Rewrite (DepthC (pi1_evalC ORELSEC pi2_evalC)) 0
| |    THEN
| |    Reduce 0
| |
| ⊢  <(n+m),(m1+m),(λj.a2(j-m))> ∈ n:ℤ×m:{n...}×{n..m⁻}→T  by Auto
| |
| |\
| | .....set predicate .....
| | 7. <n,m1,a2>.l = i
| | 8. <n,m1,a2>.u = j
| | 9. m : ℤ
| | ⊢  n+m≤m1+m  by SupInf
| |
| |\
| | 7. <n,m1,a2>.l = i
| | 8. <n,m1,a2>.u = j
| | 9. m : ℤ
| | 10. j1 : {(n+m)..(m1+m)⁻}
| | ⊢  n≤j1-m  by SupInf
| |
| .\
| . 7. <n,m1,a2>.l = i
| . 8. <n,m1,a2>.u = j
| . 9. m : ℤ
| . 10. j1 : {(n+m)..(m1+m)⁻}
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| . ⊢  j1-m<m1  by SupInf
|
.\
. .....set predicate .....
. 4. a : [T]Array
. 5. a.l = i & a.u = j
. 6. m : ℤ
. ⊢  a[++m].l = i+m & a[++m].u = j+m
. | by Analyze 4 THEN Analyze 5
. |    THEN
. |    Repeat (Unfolds [‘array_shift‘;‘array_lb‘;‘array_ub‘] 0)
. |    THEN
. |    Rewrite (DepthC (pi1_evalC ORELSEC pi2_evalC)) 0
. |
. 4. n : ℤ
. 5. m1 : {n...}
. 6. a2 : {n..m1⁻}→T
. 7. <n,m1,a2>.l = i & <n,m1,a2>.u = j
. 8. m : ℤ
. ⊢  n+m = i+m & m1+m = j+m
.   by Unfolds [‘array_ub‘;‘array_lb‘] 7
.      THEN
.      Rewrite (DepthC (pi1_evalC ORELSEC pi2_evalC)) 7
END
```