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extension_lemma  THM

⊢  ∀a,a':Assignment.
|  a' extends a ⇒ (∀x:Var. ¬a(x) = 3_1 ⇒ a(x) = a'(x))  by UnivCD
|
1. a : Assignment
2. a' : Assignment
3. a' extends a
4. x : Var
5. ¬a(x) = 3_1
⊢  a(x) = a'(x)  by Analyze 3
|
3. a'↓a = a
4. x : Var
5. ¬a(x) = 3_1
⊢   by RevHypSubst 3 -1
|
5. ¬a'↓a(x) = 3_1
⊢   by ApFunToHypEquands `z' (z(x)) ℕ_3 3
|
6. a'↓a(x) = a(x)
⊢   by MoveToConcl -1 THEN GenConcl (a(x) = n) THEN Analyze 0
|
6. n : ℕ_3
7. a(x) = n
8. a'↓a(x) = n
⊢  n = a'(x)  by ThreeInd 6
|
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| 6. a(x) = 3_0
| 7. a'↓a(x) = 3_0
| ⊢  3_0 = a'(x)  by Unfold `restriction` -1
| |
| 7. (λx.case (a(x)): 3_0 → (a'(x)); 3_1 → 3_1; 3_2 → (a'(x));)(x) = 3_0
| ⊢   by AbReduce -1
| |
| 7. case (a(x)): 3_0 → (a'(x)); 3_1 → 3_1; 3_2 → (a'(x)); = 3_0
| ⊢   by HypSubst -2 -1
| |
| |\
| | 8. z : ℕ_3
| | ⊢  case z: 3_0 → (a'(x)); 3_1 → 3_1; 3_2 → (a'(x)); ∈ ℕ_3
| |   by ThreeInd -1 THEN AbReduce 0
| |
| .\
| . 7. case 3_0: 3_0 → (a'(x)); 3_1 → 3_1; 3_2 → (a'(x)); = 3_0
| . ⊢   by AbReduce -1
|
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| 6. a(x) = 3_1
| 7. a'↓a(x) = 3_1
| ⊢  3_1 = a'(x)  by Analyze 5 THEN Trivial
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. 6. a(x) = 3_2
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. 7. a'↓a(x) = 3₂
. ⊢  3₂ = a'(x)  by Unfold `restriction` -1
. |
. 7. (λx.case (a(x)): 3₀ → (a'(x)); 3₁ → 3₁; 3₂ → (a'(x));)(x) = 3₂
. ⊢   by AbReduce -1
. |
. 7. case (a(x)): 3₀ → (a'(x)); 3₁ → 3₁; 3₂ → (a'(x)); = 3₂
. ⊢   by HypSubst -2 -1
. |
. |\
. | 8. z : ℕ₃
. | ⊢  case z: 3₀ → (a'(x)); 3₁ → 3₁; 3₂ → (a'(x)); ∈ ℕ₃
. |   by ThreeInd -1 THEN AbReduce 0
. |
. .\
. . 7. case 3₂: 3₀ → (a'(x)); 3₁ → 3₁; 3₂ → (a'(x)); = 3₂
. . ⊢   by AbReduce -1
END
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