```
auto_iso_sym  THM

⊢  ∀A1:Automata(Alph;S1), A2:Automata(Alph;S2). A1 ≡ A2 ⇒ A2 ≡ A1
| by Unfold ‘auto_iso‘ 0
|
1. Alph : Type
2. S1 : Type
3. S2 : Type
4. A1 : Automata(Alph;S1)
5. A2 : Automata(Alph;S2)
6. ∃f:(S1→S2).
|  Bij(S1; S2; f)
|  & (∀s:S1, a:Alph. f(δA1(s,a)) = δA2(f(s),a))
|  & f(InitialState(A1)) = InitialState(A2)
|  & (∀s:S1. FinalState(A1)(s) = FinalState(A2)(f(s)))
⊢  ∃f:(S2→S1).
|  Bij(S2; S1; f)
|  & (∀s:S2, a:Alph. f(δA2(s,a)) = δA1(f(s),a))
|  & f(InitialState(A2)) = InitialState(A1)
|  & (∀s:S2. FinalState(A2)(s) = FinalState(A1)(f(s)))  by Analyze 6
|
6. f : S1→S2
7. Bij(S1; S2; f)
8. ∀s:S1, a:Alph. f(δA1(s,a)) = δA2(f(s),a)
9. f(InitialState(A1)) = InitialState(A2)
10. ∀s:S1. FinalState(A1)(s) = FinalState(A2)(f(s))
⊢
| by Inst
|    Thm* ∀f:(A→B). Bij(A; B; f) ⇒ (∃g:(B→A). InvFuns(A; B; f; g))
|    {bij_imp_exists_inv}
|    [S1;S2;f]
|
11. ∃g:(S2→S1). InvFuns(S1; S2; f; g)
⊢   by Analyze 11
|
11. g : S2→S1
12. InvFuns(S1; S2; f; g)
⊢   by Witness g
|
|\
| ⊢  Bij(S2; S1; g)
| | by Inst
| |    Thm* ∀f:(A→B), g:(B→A).
| |         InvFuns(A; B; f; g) ⇒ InvFuns(B; A; g; f)
| |    {inv_funs_sym}
| |    [S1;S2;f;g]
| |
| 13. InvFuns(S2; S1; g; f)
| ⊢
|   by Inst
|      Thm* ∀f:(A→B), g:(B→A). InvFuns(A; B; f; g) ⇒ Bij(A; B; f)
|      {fun_with_inv_is_bij}
|      [S2;S1;g;f]
|
|\
```

```
| 13. s : S2
| 14. a : Alph
| ⊢  g(δA2(s,a)) = δA1(g(s),a)  by Witness8 g(s)
| |
| 8. f(InitialState(A1)) = InitialState(A2)
| 9. ∀s:S1. FinalState(A1)(s) = FinalState(A2)(f(s))
| 10. g : S2→S1
| 11. InvFuns(S1; S2; f; g)
| 12. s : S2
| 13. a : Alph
| 14. ∀a:Alph. f(δA1(g(s),a)) = δA2(f(g(s)),a)
| ⊢   by Witness14 a
| |
| 14. f(δA1(g(s),a)) = δA2(f(g(s)),a)
| ⊢   by ApFunToHypEquands 't' (g(t)) S1 14
| |
| 15. g(f(δA1(g(s),a))) = g(δA2(f(g(s)),a))
| ⊢   by RW (SweepUpC add_composeC) 15
| |
| 15. (g o f o (δA1 o g)(s))(a) = (g o (δA2 o f o g)(s))(a)
| ⊢   by Analyze 11
| |
| 11. g o f = Id
| 12. f o g = Id
| 13. s : S2
| 14. a : Alph
| 15. f(δA1(g(s),a)) = δA2(f(g(s)),a)
| 16. (g o f o (δA1 o g)(s))(a) = (g o (δA2 o f o g)(s))(a)
| ⊢   by RWH (HypC 12) 16
| |
| 16. (g o f o (δA1 o g)(s))(a) = (g o (δA2 o Id)(s))(a)
| ⊢
| | by RWH
| |    (LemmaC
| |     Thm* ∀f:(A→B), g:(B→C), h:(C→D).
| |          h o (g o f) = (h o g) o f ∈ A→D
| |     {comp_assoc})
| |    16
| |
| 16. (g o f o (δA1 o g)(s))(a) = (g o (δA2 o Id)(s))(a)
| ⊢   by RWH (HypC 11) 16
| |
| 16. (Id o (δA1 o g)(s))(a) = (g o (δA2 o Id)(s))(a)
| ⊢   by Reduce 16
|
|\
| ⊢  g(InitialState(A2)) = InitialState(A1)  by Analyze 12

| |
| 12. g o f = Id
| 13. f o g = Id
| ⊢   by ApFunToHypEquands 't' (g(t)) S1 9
| |
| 14. g(f(InitialState(A1))) = g(InitialState(A2))
| ⊢   by RWH add_composeC 14
| |
```

```
| 14. (g o f)(InitialState(A1)) = g(InitialState(A2))
| ⊢   by RWH (HypC 12) 14
| |
| 14. Id(InitialState(A1)) = g(InitialState(A2))
| ⊢   by Reduce 14
|
.\
. 13. s : S2
. ⊢  FinalState(A2)(s) = FinalState(A1)(g(s))  by Witness10 g(s)
. |
. 10. g : S2→S1
. 11. InvFuns(S1; S2; f; g)
. 12. s : S2
. 13. FinalState(A1)(g(s)) = FinalState(A2)(f(g(s)))
. ⊢   by RW (LowerC add_composeC) 13
. |
. 13. (FinalState(A1) o g)(s) = FinalState(A2)((f o g)(s))
. ⊢   by Analyze 11
. |
. 11. g o f = Id
. 12. f o g = Id
. 13. s : S2
. 14. (FinalState(A1) o g)(s) = FinalState(A2)((f o g)(s))
. ⊢   by RWH (HypC 12) 14
. |
. 14. (FinalState(A1) o g)(s) = FinalState(A2)(Id(s))
. ⊢   by Reduce 14
END
```