```
refine_ge  THM

⊢  ∀A1:Automata(Alph;S1), A2:Automata(Alph;S2).
|  Con(A1) & Con(A2) ⇒ A1 ⊆ A2  ⇒ |S1| ≥ |S2|  by Auto
|
1. Alph : Type
2. S1 : Type
3. S2 : Type
4. A1 : Automata(Alph;S1)
5. A2 : Automata(Alph;S2)
6. Con(A1)
7. Con(A2)
8. A1 ⊆ A2
⊢  |S1| ≥ |S2|  by Unfold ‘connected‘ 6
|
6. ∀s:S1. ∃l:Alph*. (Result(A1)l) = s
7. Con(A2)
8. A1 ⊆ A2
⊢
| by Inst
|    Thm* ∀Q:(A→B→Prop).
|         (∀x:A. ∃y:B. Q(x,y)) ⇒ (∃f:(A→B). ∀x:A. Q(x,f(x)))
|    {ax_choice}
|    [S1;Alph*;λ2s,l. (Result(A1)l) = s]
|
9. ∃f:(S1→Alph*). ∀x:S1. (Result(A1)f(x)) = x
⊢   by Analyze 9
|
9. f : S1→Alph*
10. ∀x:S1. (Result(A1)f(x)) = x
⊢   by Unfold ‘card_ge‘ 0
|
⊢  ∃f:(S1→S2). Surj(S1; S2; f)  by Witness λs.Result(A2)f(s)
|
⊢  Surj(S1; S2; λs.Result(A2)f(s))  by Analyze 0 THEN Reduce 0
|
11. b : S2
⊢  ∃a:S1. (Result(A2)f(a)) = b  by Unfold ‘connected‘ 7
|
7. ∀s:S2. ∃l:Alph*. (Result(A2)l) = s
8. A1 ⊆ A2
9. f : S1→Alph*
10. ∀x:S1. (Result(A1)f(x)) = x
11. b : S2
⊢   by Witness7 b
|
7. A1 ⊆ A2
8. f : S1→Alph*
9. ∀x:S1. (Result(A1)f(x)) = x
10. b : S2
11. ∃l:Alph*. (Result(A2)l) = b
⊢   by Analyze 11
|
11. l : Alph*
12. (Result(A2)l) = b
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⊢   by Witness Result(A1)l
|
⊢  (Result(A2)f(Result(A1)l)) = b  by Unfold ‘refine‘ 7
|
7. ∀x,y:Alph*.
|  (Result(A1)x) = (Result(A1)y) ⇒ (Result(A2)x) = (Result(A2)y)
8. f : S1→Alph*
9. ∀x:S1. (Result(A1)f(x)) = x
10. b : S2
11. l : Alph*
12. (Result(A2)l) = b
⊢   by Witness7 f(Result(A1)l)
|
7. f : S1→Alph*
8. ∀x:S1. (Result(A1)f(x)) = x
9. b : S2
10. l : Alph*
11. (Result(A2)l) = b
12. ∀y:Alph*.
|   (Result(A1)f(Result(A1)l)) = (Result(A1)y) ⇒
|   (Result(A2)f(Result(A1)l)) = (Result(A2)y)
⊢   by Witness12 l
|
12. (Result(A1)f(Result(A1)l)) = (Result(A1)l) ⇒
|   (Result(A2)f(Result(A1)l)) = (Result(A2)l)
⊢   by Analyze 12
|
|\
| .....antecedent .....
| 11. (Result(A2)l) = b
| ⊢  (Result(A1)f(Result(A1)l)) = (Result(A1)l)  by BackThru 8
|
.\
. 12. (Result(A2)f(Result(A1)l)) = (Result(A2)l)
. ⊢   by RWH (HypC 11) 12
END
```