```
Rl_quo_is_decidable  THM

⊢  ∀Auto:Automata(Alph;St).
|  Fin(Alph) & Fin(St) ⇒
|  (∀x,y:Alph*.
|   Dec(x = y ∈ x,y:Alph*//(x LangOf(Auto)-induced Equiv y)))
| by UnivCD
|
1. Alph : Type
2. St : Type
3. Auto : Automata(Alph;St)
4. Fin(Alph) & Fin(St)
5. x : Alph*
6. y : Alph*
⊢  Dec(x = y)
| by Inst
|    Thm* ∀Auto:Automata(Alph;St).
|         Fin(Alph) & Fin(St) ⇒
|         Fin(x,y:Alph*//(x LangOf(Auto)-induced Equiv y))
|    {mn_13}
|    [Alph;St;Auto]
|
7. Fin(x,y:Alph*//(x LangOf(Auto)-induced Equiv y))
⊢   by Unfold `finite` 7 THEN Analyze 7
|
7. n : ℕ
8. ∃f:(ℕn→(x,y:Alph*//(x LangOf(Auto)-induced Equiv y))).
|  Bij(ℕn; x,y:Alph*//(x LangOf(Auto)-induced Equiv y); f)
⊢
| by Inst
|    Thm* ∀L:LangOver(A). EquivRel x,y:A*. x L-induced Equiv y
|    {lang_rel_equi}
|    [Alph;LangOf(Auto)]
|    THEN
|    RWH
|    (LemmaC
|     Thm* (∃f:(A→B). Bij(A; B; f)) ⟺ (A ∼ B)    {bij_iff_1_1_corr})
|    8
|
8. ℕn ∼ (x,y:Alph*//(x LangOf(Auto)-induced Equiv y))
9. EquivRel x,y:Alph*. x LangOf(Auto)-induced Equiv y
⊢   by Analyze 8
|
8. f : ℕn→(x,y:Alph*//(x LangOf(Auto)-induced Equiv y))
9. ∃g:((x,y:Alph*//(x LangOf(Auto)-induced Equiv y))→ℕn).
|  InvFuns(ℕn; x,y:Alph*//(x LangOf(Auto)-induced Equiv y); f; g)
10. EquivRel x,y:Alph*. x LangOf(Auto)-induced Equiv y
⊢   by Analyze 9
|
9. g : (x,y:Alph*//(x LangOf(Auto)-induced Equiv y))→ℕn
10. InvFuns(ℕn; x,y:Alph*//(x LangOf(Auto)-induced Equiv y); f; g)
11. EquivRel x,y:Alph*. x LangOf(Auto)-induced Equiv y
⊢   by Assert (x = y ⟺ g(x) = g(y))
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| .....assertion .....
| ⊢  x = y ⟺ g(x) = g(y)  by Analyze 0 THEN Analyze 0
| |
| |\
| | 12. x = y
| | ⊢  g(x) = g(y)  by RWH (HypC 12) 0
| |
| .\
| . 12. g(x) = g(y)
| . ⊢  x = y
| . | by ApFunToHypEquands `a' (f(a))
| . |    (x,y:Alph*//(x LangOf(Auto)-induced Equiv y))
| . |    12
| . |
| . 13. f(g(x)) = f(g(y))
| . ⊢   by RWH add_composeC 13
| . |
| . 13. (f o g)(x) = (f o g)(y)
| . ⊢   by Analyze 10
| . |
| . 10. g o f = Id
| . 11. f o g = Id
| . 12. EquivRel x,y:Alph*. x LangOf(Auto)-induced Equiv y
| . 13. g(x) = g(y)
| . 14. (f o g)(x) = (f o g)(y)
| . ⊢   by RWH (HypC 11) 14 THEN Reduce 14
|
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. 12. x = y ⟺ g(x) = g(y)
. ⊢   by RWH (HypC 12) 0
. |
. ⊢  Dec(g(x) = g(y))  by ProveDecidable
END
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