```
any_ge_min_auto  THM

⊢  ∀Auto:Automata(Alph;St), S:Type, A:Automata(Alph;S).
|  Fin(Alph) ⇒
|  Fin(St) ⇒
|  LangOf(Auto) = LangOf(A) ⇒
|  Con(A) ⇒ |S| ≥ |x,y:Alph*//(x LangOf(Auto)-induced Equiv y)|
| by UnivCD
|
1. Alph : Type
2. St : Type
3. Auto : Automata(Alph;St)
4. S : Type
5. A : Automata(Alph;S)
6. Fin(Alph)
7. Fin(St)
8. LangOf(Auto) = LangOf(A)
9. Con(A)
⊢  |S| ≥ |x,y:Alph*//(x LangOf(Auto)-induced Equiv y)|
| by Inst
|    Thm* ∀L:LangOver(A). EquivRel x,y:A*. x L-induced Equiv y
|    {lang_rel_equi}
|    [Alph;LangOf(Auto)]
|    THEN
|    Inst
|    Thm* ∀L:LangOver(A). EquivRel x,y:A*. x L-induced Equiv y
|    {lang_rel_equi}
|    [Alph;LangOf(A)]
|
10. EquivRel x,y:Alph*. x LangOf(Auto)-induced Equiv y
11. EquivRel x,y:Alph*. x LangOf(A)-induced Equiv y
⊢
| by Inst
|    Thm* ∀A1:Automata(Alph;S1), A2:Automata(Alph;S2).
|         Con(A1) & Con(A2) ⇒ A1 ⊆ A2  ⇒ |S1| ≥ |S2|
|    {refine_ge}
|    [Alph
|    ;S
|    ;x,y:Alph*//(x LangOf(Auto)-induced Equiv y)
|    ;A
|    ;MinAuto(Auto)]
|
|\
| ⊢  Con(MinAuto(Auto))
|   by BackThru
|      Thm* ∀Auto:Automata(Alph;St).
|           Fin(Alph) & Fin(St) ⇒ Con(MinAuto(Auto))
|      {min_auto_con}
|
.\
. .....antecedent .....
. ⊢  A ⊆ MinAuto(Auto)
. | by Inst
. |    Thm* ∀Auto:Automata(Alph;St)
. |         , g:((x,y:Alph*//(x LangOf(Auto)-induced Equiv y))→𝔹).
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. |          Auto ⊆ A(g)
. |     {lang_auto_is_min}
. |     [Alph;S;A;λl.Auto(l)↓]
. |     THENL
. |     [Auto;Auto;Auto;Id;Id]
. |
. |\
. | .....wf .....
. | ⊢  (λl.Auto(l)↓) ∈ (x,y:Alph*//(x LangOf(A)-induced Equiv y))→𝔹
. | | by Analyze
. | |
. | |\
. | | 12. l : x,y:Alph*//(x LangOf(A)-induced Equiv y)
. | | ⊢  Auto(l)↓ ∈ 𝔹
. | | | by Unfold ‘lang_eq‘ 8 THEN Unfold ‘auto_lang‘ 8 THEN Reduce 8
. | | |
. | | 8. ∀l:Alph*. Auto(l)↓ ⟺ A(l)↓
. | | 9. Con(A)
. | | 10. EquivRel x,y:Alph*. x LangOf(Auto)-induced Equiv y
. | | 11. EquivRel x,y:Alph*. x LangOf(A)-induced Equiv y
. | | 12. l : x,y:Alph*//(x LangOf(A)-induced Equiv y)
. | | ⊢   by Analyze 12
. | | |
. | | 12. l1 : Alph*
. | | 13. l2 : Alph*
. | | 14. l1 LangOf(A)-induced Equiv l2
. | | ⊢  Auto(l1)↓ = Auto(l2)↓
. | | | by BackThru
. | | |    Thm* ∀a,b:𝔹. (a ⟺ b) ⇒ a = b    {iff_imp_equal_bool}
. | | |
. | | ⊢  Auto(l1)↓ ⟺ Auto(l2)↓  by RWH (HypC 8) 0
. | | |
. | | ⊢  A(l1)↓ ⟺ A(l2)↓
. | | | by Unfold ‘lang_rel‘ 14 THEN Reduce 14 THEN Witness14 nil
. | | |    THEN
. | | |    Reduce 14
. | | |
. | | 14. LangOf(A)(l1) ⟺ LangOf(A)(l2)
. | | ⊢   by Unfold ‘auto_lang‘ 14 THEN Reduce 14
. | |
. | .\
. | . .....eq aux .....
. | . ⊢  x,y:Alph*//(x LangOf(A)-induced Equiv y) ∈ Type  by Auto
. |
. .\
. . 12. A ⊆ A(λl.Auto(l)↓)
. . ⊢   by Unfold ‘min_auto‘ 0

. . |
. . ⊢  A ⊆ A(λl.Auto(l)↓)
. . | by Assert
. . |    (x,y:Alph*//(x LangOf(Auto)-induced Equiv y)
. . |     =
. . |     x,y:Alph*//(x LangOf(A)-induced Equiv y))
. . |
. . |\
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. . | .....assertion .....
. . | ⊢  x,y:Alph*//(x LangOf(Auto)-induced Equiv y)
. . | |  =
. . | |  x,y:Alph*//(x LangOf(A)-induced Equiv y)
. . | | by refine
. . | |    (make_primitive_rule
. . | |     RULE H  ⊢ x,y:A//E = u,v:B//F ∈ Type{i}
. . | |
. . | |            BY quotientEquality r; s; w
. . | |
. . | |            H ⊢ x,y:A//E ∈ Type{i}
. . | |            H ⊢ u,v:B//F ∈ Type{i}
. . | |            H ⊢ A = B ∈ Type{i}
. . | |            H, w:(A = B ∈ Type{i}), r:A, s:A
. . | |               ⊢ E[r,s/x,y] ⟺ F[r,s/u,v]
. . | |     {quotientEquality}
. . | |     [make_variable_argument ‘x’
. . | |     ;make_variable_argument ‘y’
. . | |     ;make_variable_argument ‘z’])
. . | |    THEN
. . | |    Unfold ‘lang_rel‘ 0
. . | |    THEN
. . | |    Unfold ‘lang_rel‘ 16
. . | |    THEN
. . | |    Reduce 0
. . | |    THEN
. . | |    Reduce 16
. . | |    THEN
. . | |    Analyze 0
. . | |    THEN
. . | |    Unfold ‘lang_eq‘ 8
. . | |
. . | |\
. . | | 8. ∀l:Alph*. LangOf(Auto)(l) ⟺ LangOf(A)(l)
. . | | 9. Con(A)
. . | | 10. EquivRel x,y:Alph*. x LangOf(Auto)-induced Equiv y
. . | | 11. EquivRel x,y:Alph*. x LangOf(A)-induced Equiv y
. . | | 12. A ⊆ A(λl.Auto(l)↓)
. . | | 13. Alph* = Alph*
. . | | 14. x : Alph*
. . | | 15. y : Alph*
. . | | 16. ∀z:Alph*. LangOf(Auto)(z @ x) ⟺ LangOf(Auto)(z @ y)
. . | | 17. z1 : Alph*
. . | | ⊢  LangOf(A)(z1 @ x) ⟺ LangOf(A)(z1 @ y)
. . | |   by RWH (RevHypC 8) 0 THEN Try (Fold ‘languages‘ 0)
. . | |      THEN
. . | |      Witness16 z1
. . | |
. . | .\
. . | . 8. ∀l:Alph*. LangOf(Auto)(l) ⟺ LangOf(A)(l)
. . | . 9. Con(A)
. . | . 10. EquivRel x,y:Alph*. x LangOf(Auto)-induced Equiv y
. . | . 11. EquivRel x,y:Alph*. x LangOf(A)-induced Equiv y
. . | . 12. A ⊆ A(λl.Auto(l)↓)
. . | . 13. Alph* = Alph*
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. . | . 14. x : Alph*
. . | . 15. y : Alph*
. . | . 16. ∀z:Alph*. LangOf(A)(z @ x) ⟺ LangOf(A)(z @ y)
. . | . 17. z1 : Alph*
. . | . ⊢  LangOf(Auto)(z1 @ x) ⟺ LangOf(Auto)(z1 @ y)
. . | .   by RWH (HypC 8) 0 THEN Try (Fold ‘languages‘ 0)
. . | .      THEN
. . | .      Witness16 z1
. . |
. . .\
. . . 13. x,y:Alph*//(x LangOf(Auto)-induced Equiv y)
. . . |   =
. . . |   x,y:Alph*//(x LangOf(A)-induced Equiv y)
. . . ⊢   by RWH (HypC 13) 0 THENL [Auto;Auto;Auto;Id;Auto;Auto;Auto]
. . . |
. . . .....wf .....
. . . ⊢  A(λl.Auto(l)↓) = A(λl.Auto(l)↓)  by Fold ‘member‘ 0
. . . |
. . . ⊢  A(λl.Auto(l)↓)
. . .    ∈ Automata(Alph;x,y:Alph*//(x LangOf(Auto)-induced Equiv y))
. . .   by Fold ‘min_auto‘ 0
END
```