```
fin_alph_list_quo  THM

⊢  ∀E:(Alph*→Alph*→Prop).
|  Fin(Alph) & (EquivRel x,y:Alph*. x E y) & (∀x,y:Alph*. Dec(x E y))
|  ⇒
|
|  (∃h:(Alph*→Alph*).
|   (∀x,y:Alph*. (x E y) ⟺ h(x) = h(y)) & (∀x:Alph*. x E (h(x))))
| by UnivCD
|
1. Alph : Type
2. E : Alph*→Alph*→Prop
3. Fin(Alph) & (EquivRel x,y:Alph*. x E y) & (∀x,y:Alph*. Dec(x E y))
⊢  ∃h:(Alph*→Alph*).
|  (∀x,y:Alph*. (x E y) ⟺ h(x) = h(y)) & (∀x:Alph*. x E (h(x)))
| by Analyze 3 THEN Analyze 4
|
3. Fin(Alph)
4. EquivRel x,y:Alph*. x E y
5. ∀x,y:Alph*. Dec(x E y)
⊢   by Analyze 3
|
3. n : ℕ
4. ∃f:(ℕn→Alph). Bij(ℕn; Alph; f)
5. EquivRel x,y:Alph*. x E y
6. ∀x,y:Alph*. Dec(x E y)
⊢
| by RWH
|    (LemmaC
|     Thm* (∃f:(A→B). Bij(A; B; f)) ⟺ (A ∼ B)    {bij_iff_1_1_corr})
|    4
|
4. ℕn ∼ Alph
5. EquivRel x,y:Alph*. x E y
6. ∀x,y:Alph*. Dec(x E y)
⊢   by Analyze 4 THEN Analyze 5
|
4. f : ℕn→Alph
5. g : Alph→ℕn
6. InvFuns(ℕn; Alph; f; g)
7. EquivRel x,y:Alph*. x E y
8. ∀x,y:Alph*. Dec(x E y)
⊢
| by Inst
|    Thm* ∀q:ℕ, E:(ℕq*→ℕq*→Prop).
|          (EquivRel x,y:ℕq*. x E y) & (∀x,y:ℕq*. Dec(x E y)) ⇒
|          (∃h:(ℕq*→ℕq*).
|           (∀x,y:ℕq*. (x E y) ⟺ h(x) = h(y)) & (∀x:ℕq*. x E (h(x))))
|    {list_quo_choice}
|    [n;λx,y. map(f;x) E map(f;y)]
|    THEN
|    Reduce 0
|
|\
| ⊢  EquivRel x,y:ℕn*. map(f;x) E map(f;y)
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| | by Unfold `equiv_rel` 0 THEN Unfold `refl` 0 THEN Unfold `trans` 0
| |    THEN
| |    Unfold `sym` 0
| |    THEN
| |    Unfold `equiv_rel` 7
| |    THEN
| |    Unfold `refl` 7
| |    THEN
| |    Unfold `trans` 7
| |    THEN
| |    Unfold `sym` 7
| |
| |\
| | 7. ∀a:Alph*. a E a
| | 8. ∀a,b:Alph*. (a E b) ⇒ (b E a)
| | 9. ∀a,b,c:Alph*. (a E b) ⇒ (b E c) ⇒ (a E c)
| | 10. ∀x,y:Alph*. Dec(x E y)
| | 11. a : ℕn*
| | ⊢  map(f;a) E map(f;a)  by BackThru 7
| |
| |\
| | 7. ∀a:Alph*. a E a
| | 8. ∀a,b:Alph*. (a E b) ⇒ (b E a)
| | 9. ∀a,b,c:Alph*. (a E b) ⇒ (b E c) ⇒ (a E c)
| | 10. ∀x,y:Alph*. Dec(x E y)
| | 11. a : ℕn*
| | 12. b : ℕn*
| | 13. map(f;a) E map(f;b)
| | ⊢  map(f;b) E map(f;a)  by FwdThru 8 [13]
| |
| .\
| . 7. ∀a:Alph*. a E a
| . 8. ∀a,b:Alph*. (a E b) ⇒ (b E a)
| . 9. ∀a,b,c:Alph*. (a E b) ⇒ (b E c) ⇒ (a E c)
| . 10. ∀x,y:Alph*. Dec(x E y)
| . 11. a : ℕn*
| . 12. b : ℕn*
| . 13. c : ℕn*
| . 14. map(f;a) E map(f;b)
| . 15. map(f;b) E map(f;c)
| . ⊢  map(f;a) E map(f;c)  by FwdThru 9 [14;15]
|
|\
| 9. x : ℕn*
| 10. y : ℕn*
| ⊢  Dec(map(f;x) E map(f;y))  by BackThru 8
|

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. 9. ∃h:(ℕn*→ℕn*).
. |  (∀x,y:ℕn*. (x (λx,y. map(f;x) E map(f;y)) y) ⟺ h(x) = h(y))
. |  & (∀x:ℕn*. x (λx,y. map(f;x) E map(f;y)) (h(x)))
. ⊢   by Analyze 9
. |
. 9. h : ℕn*→ℕn*
. 10. (∀x,y:ℕn*. (x (λx,y. map(f;x) E map(f;y)) y) ⟺ h(x) = h(y))
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. |   & (∀x:ℕn*. x (λx,y. map(f;x) E map(f;y)) (h(x)))
. ⊢   by Reduce 10
. |
. 10. (∀x,y:ℕn*. (map(f;x) E map(f;y)) ⟺ h(x) = h(y))
. |   & (∀x:ℕn*. map(f;x) E map(f;h(x)))
. ⊢   by Witness λl.map(f;h(map(g;l))) THEN Reduce 0
. |
. ⊢  (∀x,y:Alph*. (x E y) ⟺ map(f;h(map(g;x))) = map(f;h(map(g;y))))
. |  & (∀x:Alph*. x E map(f;h(map(g;x))))  by UnivCD
. |
. ⊢   by Analyze 10 THEN Analyze 0
. |
. |\
. | 10. ∀x,y:ℕn*. (map(f;x) E map(f;y)) ⟺ h(x) = h(y)
. | 11. ∀x:ℕn*. map(f;x) E map(f;h(x))
. | ⊢  ∀x,y:Alph*. (x E y) ⟺ map(f;h(map(g;x))) = map(f;h(map(g;y)))
. | | by UnivCD THEN InstHyp [map(g;x);map(g;y)] 10
. | |
. | 12. x : Alph*
. | 13. y : Alph*
. | 14. (map(f;map(g;x)) E map(f;map(g;y)))
. | |   ⟺
. | |   h(map(g;x)) = h(map(g;y))
. | ⊢  (x E y) ⟺ map(f;h(map(g;x))) = map(f;h(map(g;y)))
. | | by RWH
. | |    (LemmaC
. | |     Thm* ∀f:(A→B), g:(B→C), as:A*.
. | |          map(g;map(f;as)) = map(g o f;as)
. | |     {map_map})
. | |    14
. | |
. | 14. (map(f o g;x) E map(f o g;y)) ⟺ h(map(g;x)) = h(map(g;y))
. | ⊢   by Analyze 6
. | |
. | 6. g o f = Id
. | 7. f o g = Id
. | 8. EquivRel x,y:Alph*. x E y
. | 9. ∀x,y:Alph*. Dec(x E y)
. | 10. h : ℕn*→ℕn*
. | 11. ∀x,y:ℕn*. (map(f;x) E map(f;y)) ⟺ h(x) = h(y)
. | 12. ∀x:ℕn*. map(f;x) E map(f;h(x))
. | 13. x : Alph*
. | 14. y : Alph*
. | 15. (map(f o g;x) E map(f o g;y)) ⟺ h(map(g;x)) = h(map(g;y))
. | ⊢   by RWH (HypC 7) 15
. | |
. | 15. (map(Id;x) E map(Id;y)) ⟺ h(map(g;x)) = h(map(g;y))
. | ⊢   by RWH (LemmaC Thm* ∀as:A*. map(Id;as) = as    {map_id}) 15
. | |
. | 15. (x E y) ⟺ h(map(g;x)) = h(map(g;y))
. | ⊢   by RWH (HypC 15) 0
. | |
. | ⊢  h(map(g;x)) = h(map(g;y))
. | |  ⟺
. | |  map(f;h(map(g;x))) = map(f;h(map(g;y)))
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. | | by Analyze 0 THEN Analyze 0
. | |
. | |\
. | | 16. h(map(g;x)) = h(map(g;y))
. | | ⊢  map(f;h(map(g;x))) = map(f;h(map(g;y)))  by RWH (HypC 16) 0
. | |
. | .\
. | . 16. map(f;h(map(g;x))) = map(f;h(map(g;y)))
. | . ⊢  h(map(g;x)) = h(map(g;y))
. | . | by ApFunToHypEquands ‘z’ map(g;z) (ℕn*) 16
. | . |
. | . 17. map(g;map(f;h(map(g;x)))) = map(g;map(f;h(map(g;y))))
. | . ⊢
. | . | by RWH
. | . |    (LemmaC
. | . |     Thm* ∀f:(A→B), g:(B→C), as:A*.
. | . |          map(g;map(f;as)) = map(g o f;as)
. | . |     {map_map})
. | . |    17
. | . |
. | . 17. map(g o f;h(map(g;x))) = map(g o f;h(map(g;y)))
. | . ⊢
. | .   by RWH (HypC 6) 17
. | .      THEN
. | .      RWH (LemmaC Thm* ∀as:A*. map(Id;as) = as    {map_id}) 17
. |
. .\
. . 10. ∀x,y:ℕn*. (map(f;x) E map(f;y)) ⟺ h(x) = h(y)
. . 11. ∀x:ℕn*. map(f;x) E map(f;h(x))
. . ⊢  ∀x:Alph*. x E map(f;h(map(g;x)))
. . | by Analyze 0 THEN Witness11 map(g;x)
. . |
. . 11. x : Alph*
. . 12. map(f;map(g;x)) E map(f;h(map(g;x)))
. . ⊢  x E map(f;h(map(g;x)))
. . | by RWH
. . |    (LemmaC
. . |     Thm* ∀f:(A→B), g:(B→C), as:A*.
. . |          map(g;map(f;as)) = map(g o f;as)
. . |     {map_map})
. . |    12
. . |    THEN
. . |    Analyze 6
. . |
. . 6. g o f = Id
. . 7. f o g = Id
. . 8. EquivRel x,y:Alph*. x E y
. . 9. ∀x,y:Alph*. Dec(x E y)
. . 10. h : ℕn*→ℕn*
. . 11. ∀x,y:ℕn*. (map(f;x) E map(f;y)) ⟺ h(x) = h(y)
. . 12. x : Alph*
. . 13. map(f o g;x) E map(f;h(map(g;x)))
. . ⊢
. .   by RWH (HypC 7) 13
. .      THEN
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. .       RWH (LemmaC Thm* ∀as:A*. map(Id;as) = as     {map_id}) 13
END
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