```
homo_is_surj  THM

⊢  ∀Auto:Automata(Alph;St), c:(St→Alph*).
|  (∀q:St. (Result(Auto)c(q)) = q) & Fin(Alph) & Fin(St) ⇒
|  Surj(St; x,y:Alph*//(x LangOf(Auto)-induced Equiv y); c)
| by UnivCD
|
1. Alph : Type
2. St : Type
3. Auto : Automata(Alph;St)
4. c : St→Alph*
5. (∀q:St. (Result(Auto)c(q)) = q) & Fin(Alph) & Fin(St)
⊢  Surj(St; x,y:Alph*//(x LangOf(Auto)-induced Equiv y); c)
| by Inst
|    Thm* ∀L:LangOver(A). EquivRel x,y:A*. x L-induced Equiv y
|    {lang_rel_equi}
|    [Alph;LangOf(Auto)]
|
6. EquivRel x,y:Alph*. x LangOf(Auto)-induced Equiv y
⊢
| by Inst
|    Thm* ∀E:(Alph*→Alph*→Prop).
|         Fin(Alph)
|         & (EquivRel x,y:Alph*. x E y)
|         & (∀x,y:Alph*. Dec(x E y))
|         ⇒
|
|         (∃h:(Alph*→Alph*).
|          (∀x,y:Alph*. (x E y) ⟺ h(x) = h(y))
|          & (∀x:Alph*. x E (h(x))))
|    {fin_alph_list_quo}
|    [Alph;λx,y. x = y]
|
|\
| 5. ∀q:St. (Result(Auto)c(q)) = q
| 6. Fin(Alph)
| 7. Fin(St)
| 8. EquivRel x,y:Alph*. x LangOf(Auto)-induced Equiv y
| ⊢  EquivRel x,y:Alph*. x (λx,y. x = y) y  by Reduce 0
| |
| ⊢  EquivRel x,y:Alph*. x = y
|   by Unfold `equiv_rel` 0 THEN Unfold `refl` 0 THEN Unfold `sym` 0
|      THEN
|      Unfold `trans` 0
|
|\
| .....decidable? .....
| 5. ∀q:St. (Result(Auto)c(q)) = q
| 6. Fin(Alph)
| 7. Fin(St)
| 8. EquivRel x,y:Alph*. x LangOf(Auto)-induced Equiv y
| 9. x : Alph*
| 10. y : Alph*
| ⊢  Dec(x (λx,y. x = y) y)  by Reduce 0
| |
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| ⊢  Dec(x = y)
|   by BackThru
|      Thm* ∀Auto:Automata(Alph;St).
|           Fin(Alph) & Fin(St) ⇒
|           (∀x,y:Alph*.
|            Dec(x = y ∈ x,y:Alph*//(x LangOf(Auto)-induced Equiv y)))
|      {Rl_quo_is_decidable}
|
.\
. 7. ∃h:(Alph*→Alph*).
. |  (∀x,y:Alph*. (x (λx,y. x = y) y) ⟺ h(x) = h(y))
. |  & (∀x:Alph*. x (λx,y. x = y) (h(x)))
. ⊢   by Analyze 7
. |
. 7. h : Alph*→Alph*
. 8. (∀x,y:Alph*. (x (λx,y. x = y) y) ⟺ h(x) = h(y))
. |  & (∀x:Alph*. x (λx,y. x = y) (h(x)))
. ⊢   by Unfold ‘surject‘ 0
. |
. ⊢  ∀b:x,y:Alph*//(x LangOf(Auto)-induced Equiv y). ∃a:St. c(a) = b
. | by Analyze 0
. |
. 9. b : x,y:Alph*//(x LangOf(Auto)-induced Equiv y)
. ⊢  ∃a:St. c(a) = b  by Witness Result(Auto)h(b)
. |
. |\
. | .....wf .....
. | ⊢  (Result(Auto)h(b)) ∈ St  by Analyze THENL [Auto;Auto;Auto;Id]
. | |
. | ⊢  h(b) ∈ Alph*  by Analyze 9
. | |
. | 9. b1 : Alph*
. | 10. b2 : Alph*
. | 11. b1 LangOf(Auto)-induced Equiv b2
. | ⊢  h(b1) = h(b2)  by Analyze 8
. | |
. | 8. ∀x,y:Alph*. (x (λx,y. x = y) y) ⟺ h(x) = h(y)
. | 9. ∀x:Alph*. x (λx,y. x = y) (h(x))
. | 10. b1 : Alph*
. | 11. b2 : Alph*
. | 12. b1 LangOf(Auto)-induced Equiv b2
. | ⊢   by Reduce 8
. | |
. | 8. ∀x,y:Alph*. x = y ⟺ h(x) = h(y)
. | 9. ∀x:Alph*. x (λx,y. x = y) (h(x))
. | 10. b1 : Alph*
. | 11. b2 : Alph*
. | 12. b1 LangOf(Auto)-induced Equiv b2
. | ⊢   by RWH (RevHypC 8) 0
. | |
. | ⊢  b1 = b2  by Analyze
. |
. |\
. | ⊢  c(Result(Auto)h(b)) = b  by Analyze 9
. | |
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. | 9. b1 : Alph*
. | 10. b2 : Alph*
. | 11. b1 LangOf(Auto)-induced Equiv b2
. | ⊢  c(Result(Auto)h(b1)) = b2  by Analyze 5
. | |
. | 5. ∀q:St. (Result(Auto)c(q)) = q
. | 6. Fin(Alph) & Fin(St)
. | 7. EquivRel x,y:Alph*. x LangOf(Auto)-induced Equiv y
. | 8. h : Alph*→Alph*
. | 9. (∀x,y:Alph*. (x (λx,y. x = y) y) ⟺ h(x) = h(y))
. | |  & (∀x:Alph*. x (λx,y. x = y) (h(x)))
. | 10. b1 : Alph*
. | 11. b2 : Alph*
. | 12. b1 LangOf(Auto)-induced Equiv b2
. | ⊢   by Witness5 Result(Auto)h(b1)
. | |
. | 5. Fin(Alph) & Fin(St)
. | 6. EquivRel x,y:Alph*. x LangOf(Auto)-induced Equiv y
. | 7. h : Alph*→Alph*
. | 8. (∀x,y:Alph*. (x (λx,y. x = y) y) ⟺ h(x) = h(y))
. | |  & (∀x:Alph*. x (λx,y. x = y) (h(x)))
. | 9. b1 : Alph*
. | 10. b2 : Alph*
. | 11. b1 LangOf(Auto)-induced Equiv b2
. | 12. (Result(Auto)c(Result(Auto)h(b1))) = (Result(Auto)h(b1))
. | ⊢   by Reduce 8
. | |
. | 8. (∀x,y:Alph*. x = y ⟺ h(x) = h(y)) & (∀x:Alph*. x = h(x))
. | 9. b1 : Alph*
. | 10. b2 : Alph*
. | 11. b1 LangOf(Auto)-induced Equiv b2
. | 12. (Result(Auto)c(Result(Auto)h(b1))) = (Result(Auto)h(b1))
. | ⊢   by Analyze 8 THEN InstHyp [b1;b2] 8
. | |
. | 8. ∀x,y:Alph*. x = y ⟺ h(x) = h(y)
. | 9. ∀x:Alph*. x = h(x)
. | 10. b1 : Alph*
. | 11. b2 : Alph*
. | 12. b1 LangOf(Auto)-induced Equiv b2
. | 13. (Result(Auto)c(Result(Auto)h(b1))) = (Result(Auto)h(b1))
. | 14. b1 = b2 ⟺ h(b1) = h(b2)
. | ⊢   by Analyze 14
. | |
. | 14. b1 = b2 ⇒ h(b1) = h(b2)
. | 15. (b1 = b2) ⇐ (h(b1) = h(b2))
. | ⊢   by RW (AddrC [3;2] (HypC 14)) 13
. | |
. | |\
. | | 5. Fin(Alph)
. | | 6. Fin(St)
. | | 7. EquivRel x,y:Alph*. x LangOf(Auto)-induced Equiv y
. | | 8. h : Alph*→Alph*
. | | 9. ∀x,y:Alph*. x = y ⟺ h(x) = h(y)
. | | 10. ∀x:Alph*. x = h(x)
. | | 11. b1 : Alph*
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. | | 12. b2 : Alph*
. | | 13. b1 LangOf(Auto)-induced Equiv b2
. | | 14. (Result(Auto)c(Result(Auto)h(b1))) = (Result(Auto)h(b1))
. | | 15. b1 = b2 ⇒ h(b1) = h(b2)
. | | 16. (b1 = b2) ⇐ (h(b1) = h(b2))
. | | ⊢  b1 = b2
. | | | by Inst
. | | |    Thm* ∀Auto:Automata(Alph;St)
. | | |         , g:((x,y:Alph*//(x LangOf(Auto)-induced Equiv y))→
. | | |             𝔹). Auto ⊆ A(g)
. | | |    {lang_auto_is_min}
. | | |    [Alph
. | | |    ;St
. | | |    ;Auto
. | | |    ;λl:x,y:Alph*//(x LangOf(Auto)-induced Equiv y). true₂]
. | | |
. | | 17. Auto ⊆ A(λl:x,y:Alph*//.... true₂)
. | | ⊢   by Analyze
. | |
. | .\
. | . 13. (Result(Auto)c(Result(Auto)h(b1))) = (Result(Auto)h(b2))
. | . 14. b1 = b2 ⇒ h(b1) = h(b2)
. | . 15. (b1 = b2) ⇐ (h(b1) = h(b2))
. | . ⊢
. | . | by Inst
. | . |    Thm* ∀Auto:Automata(Alph;St)
. | . |         , g:((x,y:Alph*//(x LangOf(Auto)-induced Equiv y))→
. | . |             𝔹). Auto ⊆ A(g)
. | . |    {lang_auto_is_min}
. | . |    [Alph
. | . |    ;St
. | . |    ;Auto
. | . |    ;λl:x,y:Alph*//(x LangOf(Auto)-induced Equiv y). true₂]
. | . |
. | . 16. Auto ⊆ A(λl:x,y:Alph*//.... true₂)
. | . ⊢   by Unfold ‘refine‘ 16
. | . |

. | . 16. ∀x,y:Alph*.
. | . |   (Result(Auto)x) = (Result(Auto)y) ⇒
. | . |   (Result(A(λl:x,y:Alph*//.... true₂))x)
. | . |   =
. | . |   (Result(A(λl:x,y:Alph*//.... true₂))y)
. | . ⊢   by FwdThru 16 [13]
. | . |
. | . 17. (Result(A(λl:x,y:Alph*//.... true₂))c(Result(Auto)h(b1)))
. | . |   =
. | . |   (Result(A(λl:x,y:Alph*//.... true₂))h(b2))
. | . ⊢
. | . | by RWH
. | . |    (LemmaC
. | . |     Thm* ∀L:LangOver(Alph)
. | . |          , g:((x,y:Alph*//(x L-induced Equiv y))→𝔹), l:Alph*.
. | . |          (Result(A(g))l)
. | . |          =
. | . |          l
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. | . |          ∈ x,y:Alph*//(x L-induced Equiv y)
. | . |     {lang_auto_compute})
. | . |    17
. | . |
. | . 17. c(Result(Auto)h(b1)) = h(b2)
. | . ⊢   by RWH (HypC 17) 0
. | . |
. | . ⊢  h(b2) = b2  by RWH (RevHypC 9) 0
. |
. .\
. . .....wf .....
. . 10. a : St
. . ⊢  c(a) = b ∈ Prop  by Auto
END
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