```
list_quo_choice  THM

⊢  ∀q:ℕ, E:(ℕq*→ℕq*→Prop).
|  (EquivRel x,y:ℕq*. x E y) & (∀x,y:ℕq*. Dec(x E y)) ⇒
|  (∃h:(ℕq*→ℕq*).
|   (∀x,y:ℕq*. (x E y) ⟺ h(x) = h(y)) & (∀x:ℕq*. x E (h(x))))
| by Analyze 0 THEN Decide (0<q)
|
|\
| 1. q : ℕ
| 2. 0<q
| ⊢  ∀E:(ℕq*→ℕq*→Prop).
|    (EquivRel x,y:ℕq*. x E y) & (∀x,y:ℕq*. Dec(x E y)) ⇒
|    (∃h:(ℕq*→ℕq*).
|     (∀x,y:ℕq*. (x E y) ⟺ h(x) = h(y)) & (∀x:ℕq*. x E (h(x))))
|   by Inst
|      Thm* ∀q:ℕ+, E:(ℕq*→ℕq*→Prop).
|           (EquivRel x,y:ℕq*. x E y) & (∀x,y:ℕq*. Dec(x E y)) ⇒
|           (∃h:(ℕq*→ℕq*).
|            (∀x,y:ℕq*. (x E y) ⟺ h(x) = h(y))
|            & (∀x:ℕq*. x E (h(x))))
|      {list_quo_choice_pls}
|      [q]
|
.\
. 1. q : ℕ
. 2. ¬0<q
. ⊢  ∀E:(ℕq*→ℕq*→Prop).
. |  (EquivRel x,y:ℕq*. x E y) & (∀x,y:ℕq*. Dec(x E y)) ⇒
. |  (∃h:(ℕq*→ℕq*).
. |   (∀x,y:ℕq*. (x E y) ⟺ h(x) = h(y)) & (∀x:ℕq*. x E (h(x))))
. | by UnivCD
. |
. 3. E : ℕq*→ℕq*→Prop
. 4. (EquivRel x,y:ℕq*. x E y) & (∀x,y:ℕq*. Dec(x E y))
. ⊢  ∃h:(ℕq*→ℕq*).
. |  (∀x,y:ℕq*. (x E y) ⟺ h(x) = h(y)) & (∀x:ℕq*. x E (h(x)))
. | by Witness Id
. |
. ⊢  (∀x,y:ℕq*. (x E y) ⟺ Id(x) = Id(y) ∈ ℕq*)
. |  & (∀x:ℕq*. x E (Id(x)))  by UnivCD
. |
. ⊢   by Assert (q = 0)
. |
. 5. q = 0
. ⊢   by Analyze 0
. |
. |\
. | ⊢  ∀x,y:ℕq*. (x E y) ⟺ Id(x) = Id(y)  by UnivCD
. | |
. | 6. x : ℕq*
. | 7. y : ℕq*
. | ⊢  (x E y) ⟺ Id(x) = Id(y)
. | | by Reduce 0
. | |    THEN
```

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. | |    RWH
. | |    (LemmaC
. | |     Thm* ∀q:ℕ, x:ℕq*. q = 0 ⇒ x = nil    {empty_alph_list})
. | |    0
. | |
. | ⊢  (nil E nil) ⟺ nil = nil
. | | by Analyze 4 THEN Unfold ‘equiv_rel‘ 4
. | |
. | 4. Refl(ℕq*;x,y.x E y) & Sym x,y:ℕq*. x E y & Trans x,y:ℕq*. x E y
. | 5. ∀x,y:ℕq*. Dec(x E y)
. | 6. q = 0
. | 7. ℕq*
. | 8. ℕq*
. | ⊢   by Analyze 4 THEN Unfold ‘refl‘ 4
. | |
. | 4. ∀a:ℕq*. a E a
. | 5. Sym x,y:ℕq*. x E y & Trans x,y:ℕq*. x E y
. | 6. ∀x,y:ℕq*. Dec(x E y)
. | 7. q = 0
. | 8. ℕq*
. | 9. ℕq*
. | ⊢   by Witness4 nil
. |
. .\
. . ⊢  ∀x:ℕq*. x E (Id(x))
. .   by Reduce 0 THEN Analyze 4 THEN Unfold ‘equiv_rel‘ 4
. .      THEN
. .      Analyze 4
. .      THEN
. .      Unfold ‘refl‘ 4
END
```