```
min_auto_con  THM

⊢  ∀Auto:Automata(Alph;St). Fin(Alph) & Fin(St) ⇒ Con(MinAuto(Auto))
| by UnivCD
|
1. Alph : Type
2. St : Type
3. Auto : Automata(Alph;St)
4. Fin(Alph) & Fin(St)
⊢  Con(MinAuto(Auto))
| by Inst
|    Thm* ∀L:LangOver(A). EquivRel x,y:A*. x L-induced Equiv y
|    {lang_rel_equi}
|    [Alph;LangOf(Auto)]
|
5. EquivRel x,y:Alph*. x LangOf(Auto)-induced Equiv y
⊢   by Unfold ‘connected‘ 0 THEN UnivCD
|
6. s : x,y:Alph*//(x LangOf(Auto)-induced Equiv y)
⊢  ∃l:Alph*. (Result(MinAuto(Auto))l) = s
| by Inst
|    Thm* ∀E:(Alph*→Alph*→Prop).
|         Fin(Alph)
|         & (EquivRel x,y:Alph*. x E y)
|         & (∀x,y:Alph*. Dec(x E y))
|         ⇒
|
|         (∃h:(Alph*→Alph*).
|          (∀x,y:Alph*. (x E y) ⟺ h(x) = h(y))
|          & (∀x:Alph*. x E (h(x))))
|    {fin_alph_list_quo}
|    [Alph;λx,y. x = y]
|
|\
| 4. Fin(Alph)
| 5. Fin(St)
| 6. EquivRel x,y:Alph*. x LangOf(Auto)-induced Equiv y
| 7. x,y:Alph*//(x LangOf(Auto)-induced Equiv y)
| ⊢  EquivRel x,y:Alph*. x (λx,y. x = y) y
|   by Reduce 0 THEN Unfold ‘equiv_rel‘ 0 THEN Unfold ‘refl‘ 0
|      THEN
|      Unfold ‘sym‘ 0
|      THEN
|      Unfold ‘trans‘ 0
|
|\
| .....decidable? .....
| 4. Fin(Alph)
| 5. Fin(St)
| 6. EquivRel x,y:Alph*. x LangOf(Auto)-induced Equiv y
| 7. x,y:Alph*//(x LangOf(Auto)-induced Equiv y)
| 8. x : Alph*
| 9. y : Alph*
| ⊢  Dec(x (λx,y. x = y) y)
|   by Reduce 0
```

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|      THEN
|      BackThru
|      Thm* ∀Auto:Automata(Alph;St).
|           Fin(Alph) & Fin(St) ⇒
|           (∀x,y:Alph*.
|            Dec(x = y ∈ x,y:Alph*//(x LangOf(Auto)-induced Equiv y)))
|      {Rl_quo_is_decidable}
|
.\
. 7. ∃h:(Alph*→Alph*).
. |  (∀x,y:Alph*. (x (λx,y. x = y) y) ⟺ h(x) = h(y))
. |  & (∀x:Alph*. x (λx,y. x = y) (h(x)))
. ⊢   by Reduce 7 THEN Analyze 7 THEN Analyze 8
. |
. 7. h : Alph*→Alph*
. 8. ∀x,y:Alph*. x = y ⟺ h(x) = h(y)
. 9. ∀x:Alph*. x = h(x)
. ⊢   by Witness h(s)
. |
. |\
. | .....wf .....
. | ⊢  h(s) ∈ Alph*  by Analyze 6
. | |
. | 6. h : Alph*→Alph*
. | 7. ∀x,y:Alph*. x = y ⟺ h(x) = h(y)
. | 8. ∀x:Alph*. x = h(x)
. | 9. s1 : Alph*
. | 10. s2 : Alph*
. | 11. s1 LangOf(Auto)-induced Equiv s2
. | ⊢  h(s1) = h(s2)  by RWH (RevHypC 7) 0
. | |
. | ⊢  s1 = s2  by Analyze
. |
. |\
. | ⊢  (Result(MinAuto(Auto))h(s)) = s  by Analyze 6
. | |
. | 6. h : Alph*→Alph*
. | 7. ∀x,y:Alph*. x = y ⟺ h(x) = h(y)
. | 8. ∀x:Alph*. x = h(x)
. | 9. s1 : Alph*
. | 10. s2 : Alph*
. | 11. s1 LangOf(Auto)-induced Equiv s2
. | ⊢  (Result(MinAuto(Auto))h(s1)) = s2
. | | by RW (AddrC [3] (HypC 8)) 0
. | |
. | ⊢  (Result(MinAuto(Auto))h(s1)) = h(s2)
. | | by RWH
. | |    (LemmaC
. | |     Thm* ∀Auto:Automata(Alph;St), l:Alph*.
. | |          (Result(MinAuto(Auto))l)
. | |          =
. | |          l
. | |          ∈ x,y:Alph*//(x LangOf(Auto)-induced Equiv y)
. | |     {min_auto_comp})
. | |    0
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. | |
. | ⊢  h(s1) = h(s2)  by InstHyp [s1;s2] 7
. | |
. | 12. s1 = s2 ⟺ h(s1) = h(s2)
. | ⊢   by Analyze 12
. | |
. | 12. s1 = s2 ⇒ h(s1) = h(s2)
. | 13. (s1 = s2) ⇐ (h(s1) = h(s2))
. | ⊢   by Analyze 12
. | |
. | |\
. | | .....antecedent .....
. | | 12. (s1 = s2) ⇐ (h(s1) = h(s2))
. | | ⊢  s1 = s2  by Analyze
. | |
. | .\
. | . 12. (s1 = s2) ⇐ (h(s1) = h(s2))
. | . 13. h(s1) = h(s2)
. | . ⊢   by Analyze
. | . |
. | . ⊢  (h(s1)) LangOf(Auto)-induced Equiv (h(s2))
. | . | by RWH (HypC 13) 0
. | . |
. | . ⊢  (h(s2)) LangOf(Auto)-induced Equiv (h(s2))  by Analyze 5
. | . |
. | . 5. Refl(Alph*;x,y.x LangOf(Auto)-induced Equiv y)
. | . 6. Sym x,y:Alph*. x LangOf(Auto)-induced Equiv y
. | .    & Trans x,y:Alph*. x LangOf(Auto)-induced Equiv y
. | . 7. h : Alph*→Alph*
. | . 8. ∀x,y:Alph*. x = y ⟺ h(x) = h(y)
. | . 9. ∀x:Alph*. x = h(x)
. | . 10. s1 : Alph*
. | . 11. s2 : Alph*
. | . 12. s1 LangOf(Auto)-induced Equiv s2
. | . 13. (s1 = s2) ⇐ (h(s1) = h(s2))
. | . 14. h(s1) = h(s2)
. | . ⊢   by Unfold ‘refl‘ 5 THEN BackThru 5
. |
. .\
. . .....wf .....
. . 10. l : Alph*
. . ⊢  (Result(MinAuto(Auto))l) = s ∈ Prop  by Auto
END
```