```
min_auto_sound  THM

⊢  ∀Auto:Automata(Alph;St). LangOf(Auto) = LangOf(MinAuto(Auto))
| by Unfold ‘auto_lang‘ 0 THEN Unfold ‘lang_eq‘ 0
|    THEN
|    Unfold ‘min_auto‘ 0
|    THEN
|    UnivCD
|    THEN
|    Reduce 0
|
1. Alph : Type
2. St : Type
3. Auto : Automata(Alph;St)
4. l : Alph*
⊢  Auto(l)↓ ⟺ A(λl.Auto(l)↓)(l)↓
| by RWH (AddrC [2;1] (UnfoldC ‘accept_list‘)) 0 THEN Reduce 0
|
⊢  Auto(l)↓
|  ⟺
|  FinalState(Auto)
|  (Result(Auto)Result(A(λl.FinalState(Auto)(Result(Auto)l)))l)
| by Inst
|    Thm* ∀L:LangOver(A). EquivRel x,y:A*. x L-induced Equiv y
|    {lang_rel_equi}
|    [Alph;LangOf(Auto)]
|    THEN
|    Inst
|    Thm* ∀L:LangOver(Alph), g:((x,y:Alph*//(x L-induced Equiv y))→ℙ)
|         , l:Alph*.
|         (Result(A(g))l) = l ∈ x,y:Alph*//(x L-induced Equiv y)
|    {lang_auto_compute}
|    [Alph;LangOf(Auto);λl.FinalState(Auto)(Result(Auto)l)]
|    THENL
|    [Auto;Auto;Id;Id]
|
|\
| .....wf .....
| 5. EquivRel x,y:Alph*. x LangOf(Auto)-induced Equiv y
| ⊢  (λl.FinalState(Auto)(Result(Auto)l))
| |  ∈ (x,y:Alph*//(x LangOf(Auto)-induced Equiv y))→ℙ  by Analyze
| |
| 6. l1 : x,y:Alph*//(x LangOf(Auto)-induced Equiv y)
| ⊢  FinalState(Auto)(Result(Auto)l1) ∈ ℙ  by Analyze 6
| |
| 6. l2 : Alph*
| 7. l3 : Alph*
| 8. l2 LangOf(Auto)-induced Equiv l3
| ⊢  FinalState(Auto)(Result(Auto)l2)
| |  =
| |  FinalState(Auto)(Result(Auto)l3)
| | by Unfold ‘lang_rel‘ 8 THEN Reduce 8 THEN Witness8 nil
| |    THEN
| |    Reduce 8
| |
```

```
| 8. LangOf(Auto)(l2) ⟺ LangOf(Auto)(l3)
| ⊢   by Unfold ‘auto_lang‘ 8 THEN Reduce 8
| |
| 8. Auto(l2)↓ ⟺ Auto(l3)↓
| ⊢   by Unfold ‘accept_list‘ 8
| |
| 8. FinalState(Auto)(Result(Auto)l2)
|    ⟺
|    FinalState(Auto)(Result(Auto)l3)
| ⊢
|   by BackThru
|      Thm* ∀a,b:𝔹. (a ⟺ b) ⇒ a = b    {iff_imp_equal_bool}
|
.\
. 5. EquivRel x,y:Alph*. x LangOf(Auto)-induced Equiv y
. 6. ∀l:Alph*. (Result(A(λl.FinalState(Auto)(Result(Auto)l)))l) = l
. ⊢   by Witness6 l
. |
. 6. (Result(A(λl.FinalState(Auto)(Result(Auto)l)))l) = l
. ⊢   by HypSubst 6 0
. |
. |\
. | ⊢  Auto(l)↓ ⟺ FinalState(Auto)(Result(Auto)l)
. |   by Fold ‘accept_list‘ 0
. |
. .\
. . .....wf .....
. . 7. z : x,y:Alph*//(x LangOf(Auto)-induced Equiv y)
. . ⊢  (Auto(l)↓ ⟺ FinalState(Auto)(Result(Auto)z)) ∈ Prop
. . | by Analyze THENL [Auto;Analyze]
. . |
. . ⊢  FinalState(Auto)(Result(Auto)z) ∈ 𝔹  by Fold ‘accept_list‘ 0
. . |
. . ⊢  Auto(z)↓ ∈ 𝔹  by Analyze 7
. . |
. . 7. z1 : Alph*
. . 8. z2 : Alph*
. . 9. z1 LangOf(Auto)-induced Equiv z2
. . ⊢  Auto(z1)↓ = Auto(z2)↓  by Unfold ‘lang_rel‘ 9 THEN Reduce 9
. . |
. . 9. ∀z:Alph*. LangOf(Auto)(z @ z1) ⟺ LangOf(Auto)(z @ z2)
. . ⊢   by Witness9 nil THEN Reduce 9
. . |
. . 9. LangOf(Auto)(z1) ⟺ LangOf(Auto)(z2)
. . ⊢   by Unfold ‘auto_lang‘ 9 THEN Reduce 9
. . |
. . 9. Auto(z1)↓ ⟺ Auto(z2)↓

. . ⊢
. .   by BackThru
. .      Thm* ∀a,b:𝔹. (a ⟺ b) ⇒ a = b    {iff_imp_equal_bool}
END
```