surj_is_inj THM

```
⊢  ∀n:ℕ, f:(ℕn→ℕn). Surj(ℕn; ℕn; f) ⇒ Inj(ℕn; ℕn; f)
| by Unfold ‘inject‘ 0 THEN UnivCD
|
1. n : ℕ
2. f : ℕn→ℕn
3. Surj(ℕn; ℕn; f)
4. a1 : ℕn
5. a2 : ℕn
6. f(a1) = f(a2)
⊢  a1 = a2  by Decide (a1 = a2)
|
|\
| 7. a1 = a2
| ⊢   by Auto
|
.\
. 7. ¬a1 = a2
. ⊢   by Assert (∃h:(ℕn→ℕ(n+1)). Surj(ℕn; ℕ(n+1); h))
. |
. |\
. | .....assertion .....
. | ⊢  ∃h:(ℕn→ℕ(n+1)). Surj(ℕn; ℕ(n+1); h)
. | | by Witness λx.if a2=₂x→ n else f(x) fi
. | |
. | ⊢  Surj(ℕn; ℕ(n+1); λx.if a2=₂x→ n else f(x) fi)
. | | by Unfold ‘surject‘ 0 THEN UnivCD
. | |
. | 8. b : ℕ(n+1)
. | ⊢  ∃a:ℕn. (λx.if a2=₂x→ n else f(x) fi)(a) = b
. | | by Reduce 0 THEN Decide (b = n)
. | |
. | |\
. | | 9. b = n
. | | ⊢  ∃a:ℕn. if a2=₂a→ n else f(a) fi = b
. | |   by Witness a2 THEN SplitOnConclITE
. | |
. | .\
. | . 9. ¬b = n
. | . ⊢  ∃a:ℕn. if a2=₂a→ n else f(a) fi = b  by Unfold ‘surject‘ 3
. | . |
. | . 3. ∀b:ℕn. ∃a:ℕn. f(a) = b
. | . 4. a1 : ℕn
. | . 5. a2 : ℕn
. | . 6. f(a1) = f(a2)
. | . 7. ¬a1 = a2
. | . 8. b : ℕ(n+1)
. | . 9. ¬b = n
. | . ⊢   by Witness3 b
. | . |
. | . |\
. | . | 3. a1 : ℕn
. | . | 4. a2 : ℕn
. | . | 5. f(a1) = f(a2)
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. | . | 6. ¬a1 = a2
. | . | 7. b : ℕ(n+1)
. | . | 8. ¬b = n
. | . | ⊢  b<n  by Auto
. | . |
. | . .\
. | . . 3. a1 : ℕn
. | . . 4. a2 : ℕn
. | . . 5. f(a1) = f(a2)
. | . . 6. ¬a1 = a2
. | . . 7. b : ℕ(n+1)
. | . . 8. ¬b = n
. | . . 9. ∃a:ℕn. f(a) = b
. | . . ⊢   by Analyze 9
. | . . |
. | . . 9. a : ℕn
. | . . 10. f(a) = b
. | . . ⊢   by Decide (a = a2)
. | . . |
. | . . |\
. | . . | 11. a = a2
. | . . | ⊢   by Witness a1
. | . . | |
. | . . | ⊢  if a2=_2a1→ n else f(a1) fi = b  by SplitOnConclITE
. | . . | |
. | . . | |\
. | . . | | .....truecase .....
. | . . | | 12. a2 = a1
. | . . | | ⊢  n = b  by Auto
. | . . | |
. | . . | .\
. | . . | . .....falsecase .....
. | . . | . 12. ¬a2 = a1
. | . . | . ⊢  f(a1) = b  by RWH (HypC 11) 10
. | . . |
. | . . .\
. | . . . 11. ¬a = a2
. | . . . ⊢   by Witness a THEN SplitOnConclITE
. |
. .\
. . 8. ∃h:(ℕn→ℕ(n+1)). Surj(ℕn; ℕ(n+1); h)
. . ⊢   by Analyze 8
. . |
. . 8. h : ℕn→ℕ(n+1)
. . 9. Surj(ℕn; ℕ(n+1); h)
. . ⊢   by Unfold ‘surject‘ 9
. . |
. . 9. ∀b:ℕ(n+1). ∃a:ℕn. h(a) = b
. . ⊢
. . | by FwdThru
. . |    Thm* ∀Q:(A→B→Prop).
. . |         (∀x:A. ∃y:B. Q(x,y)) ⇒ (∃f:(A→B). ∀x:A. Q(x,f(x)))
. . |    {ax_choice}
. . |    [9]
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. . 10. ∃f:(ℕ(n+1)→ℕn). ∀b:ℕ(n+1). h(f(b)) = b
. . ⊢   by Analyze 10
. . |
. . 10. f1 : ℕ(n+1)→ℕn
. . 11. ∀b:ℕ(n+1). h(f1(b)) = b
. . ⊢   by Decide (n = 0)
. . |
. . |\
. . | 12. n = 0
. . | ⊢   by Assert (f1(0) ∈ ℕ0)
. . | |
. . | 13. f1(0) ∈ ℕ0
. . | ⊢   by Analyze 13
. . |
. . .\
. . . 12. ¬n = 0
. . . ⊢
. . . | by Inst
. . . |    Thm* ∀n:{1...}, f:(ℕ(n+1)→ℕn).
. . . |          ∃i:ℕ(n+1), j:ℕi. f(i) = f(j)
. . . |    {phole_aux}
. . . |    [n;f1]
. . . |
. . . 13. ∃i:ℕ(n+1), j:ℕi. f1(i) = f1(j)
. . . ⊢   by Analyze 13 THEN Analyze 14
. . . |
. . . 13. i : ℕ(n+1)
. . . 14. j : ℕi
. . . 15. f1(i) = f1(j)
. . . ⊢   by ApFunToHypEquands ‘z’ (h(z)) ℕ(n+1) 15
. . . |
. . . 16. h(f1(i)) = h(f1(j))
. . . ⊢   by RWH (HypC 11) 16
END
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