```
surj_is_inj_gen  THM

⊢  ∀f:(A→B). (A ∼ B) & Fin(B) ⇒ Surj(A; B; f) ⇒ Inj(A; B; f)
| by Unfold ‘one_one_corr‘ 0 THEN Unfold ‘finite‘ 0 THEN UnivCD
|
1. A : Type
2. B : Type
3. f : A→B
4. (∃f:(A→B), g:(B→A). InvFuns(A; B; f; g))
|  & (∃n:ℕ, f:(ℕn→B). Bij(ℕn; B; f))
5. Surj(A; B; f)
⊢  Inj(A; B; f)
| by Analyze 4 THEN Analyze 4 THEN Analyze 6 THEN Analyze 5
|
4. f1 : A→B
5. g : B→A
6. InvFuns(A; B; f1; g)
7. n : ℕ
8. ∃f:(ℕn→B). Bij(ℕn; B; f)
9. Surj(A; B; f)
⊢
| by RWH
|    (LemmaC
|     Thm* (∃f:(A→B). Bij(A; B; f)) ⟺ (A ∼ B)    {bij_iff_1_1_corr})
|    8
|
8. ℕn ∼ B
9. Surj(A; B; f)
⊢   by Unfold ‘one_one_corr‘ 8 THEN Analyze 8 THEN Analyze 9
|
8. f2 : ℕn→B
9. g1 : B→ℕn
10. InvFuns(ℕn; B; f2; g1)
11. Surj(A; B; f)
⊢
| by Inst
|    Thm* ∀n:ℕ, f:(ℕn→ℕn). Surj(ℕn; ℕn; f) ⇒ Inj(ℕn; ℕn; f)
|    {surj_is_inj}
|    [n;g1 o f o g o f2]
|
|\
| .....antecedent .....
| ⊢  Surj(ℕn; ℕn; g1 o f o g o f2)
| | by Unfold ‘surject‘ 0 THEN Analyze 0
| |
| 12. b : ℕn
| ⊢  ∃a:ℕn. (g1 o f o g o f2)(a) = b
| | by Unfold ‘surject‘ 11 THEN Witness11 f2(b)
| |
| 11. b : ℕn
| 12. ∃a:A. f(a) = f2(b)
| ⊢   by Analyze 12
| |
| 12. a : A
| 13. f(a) = f2(b)
```

```
| ⊢   by Witness (g1 o f1)(a)
| |
| ⊢  (g1 o f o g o f2)((g1 o f1)(a)) = b  by RWH add_composeC 0
| |
| ⊢  (g1 o f o g o f2 o g1 o f1)(a) = b
| | by RWH
| |    (RevLemmaC
| |     Thm* ∀f:(A→B), g:(B→C), h:(C→D).
| |          h o (g o f) = (h o g) o f ∈ A→D
| |     {comp_assoc})
| |    0
| |
| ⊢  (g1 o f o g o f2 o g1 o f1)(a) = b
| | by RW
| |    (AddrC [2;1;2]
| |     (LemmaC
| |      Thm* ∀f:(A→B), g:(B→C), h:(C→D).
| |           h o (g o f) = (h o g) o f ∈ A→D
| |      {comp_assoc}))
| |    0
| |
| ⊢  (g1 o f o g o f2 o g1 o f1)(a) = b
| | by RW
| |    (AddrC [2;1;2;1]
| |     (RevLemmaC
| |      Thm* ∀f:(A→B), g:(B→C), h:(C→D).
| |           h o (g o f) = (h o g) o f ∈ A→D
| |      {comp_assoc}))
| |    0
| |
| ⊢  (g1 o f o g o f2 o g1 o f1)(a) = b  by Analyze 10
| |
| 10. g1 o f2 = Id
| 11. f2 o g1 = Id
| 12. b : ℕn
| 13. a : A
| 14. f(a) = f2(b)
| ⊢   by RWH (HypC 11) 0 THEN Reduce 0
| |
| ⊢  g1(f(g(f1(a)))) = b  by Analyze 6
| |
| 6. g o f1 = Id
| 7. f1 o g = Id
| 8. n : ℕ
| 9. f2 : ℕn→B
| 10. g1 : B→ℕn
| 11. g1 o f2 = Id
| 12. f2 o g1 = Id
| 13. b : ℕn
| 14. a : A
| 15. f(a) = f2(b)
| ⊢   by RW (AddrC [2;2;2] add_composeC) 0
| |
| ⊢  g1(f((g o f1)(a))) = b  by RWH (HypC 6) 0 THEN Reduce 0
| |
```

```
| ⊢  g1(f(a)) = b  by ApFunToHypEquands ‘z’ (g1(z)) ℕn 15
| |
| 16. g1(f(a)) = g1(f2(b))
| ⊢   by RW (AddrC [3] add_composeC) 16
| |
| 16. g1(f(a)) = (g1 o f2)(b)
| ⊢   by (RWH (HypC 11) 16) THEN (Reduce 16)
|
.\
. 12. Inj(ℕn; ℕn; g1 o f o g o f2)
. ⊢   by Analyze 0 THEN Analyze 0
. |
. 13. a1 : A
. 14. a2 : A
. ⊢  f(a1) = f(a2) ⇒ a1 = a2  by Analyze 0
. |
. 15. f(a1) = f(a2)
. ⊢  a1 = a2  by Unfold ‘inject‘ 12
. |
. 12. ∀a1,a2:ℕn.
. |   (g1 o f o g o f2)(a1) = (g1 o f o g o f2)(a2) ⇒ a1 = a2
. 13. a1 : A
. 14. a2 : A
. 15. f(a1) = f(a2)
. ⊢   by Witness12 g1(f1(a1)) THEN Witness15 g1(f1(a2))
. |
. 12. a1 : A
. 13. a2 : A
. 14. f(a1) = f(a2)
. 15. (g1 o f o g o f2)(g1(f1(a1))) = (g1 o f o g o f2)(g1(f1(a2)))
. |   ⇒
. |   g1(f1(a1)) = g1(f1(a2))
. ⊢   by Analyze 15
. |
. |\
. | .....antecedent .....
. | 14. f(a1) = f(a2)
. | ⊢  (g1 o f o g o f2)(g1(f1(a1))) = (g1 o f o g o f2)(g1(f1(a2)))
. | | by RWH add_composeC 0
. | |
. | ⊢  (g1 o f o g o f2 o g1)(f1(a1)) = (g1 o f o g o f2 o g1)(f1(a2))
. | | by RWH
. | |    (RevLemmaC
. | |     Thm* ∀f:(A→B), g:(B→C), h:(C→D).
. | |          h o (g o f) = (h o g) o f ∈ A→D
. | |     {comp_assoc})
. | |    0
. | |
. | ⊢  (g1 o f o g o f2 o g1)(f1(a1)) = (g1 o f o g o f2 o g1)(f1(a2))
. | | by RW
. | |    (AddrC [2;1;2]
. | |     (RevLemmaC
. | |      Thm* ∀f:(A→B), g:(B→C), h:(C→D).
. | |           h o (g o f) = (h o g) o f ∈ A→D
. | |      {comp_assoc}))
```

```
. | |    0
. | |    THEN
. | |    RW
. | |    (AddrC [3;1;2]
. | |     (RevLemmaC
. | |      Thm* ∀f:(A→B), g:(B→C), h:(C→D).
. | |           h o (g o f) = (h o g) o f ∈ A→D
. | |      {comp_assoc}))
. | |    0
. | |
. | ⊢  (g1 o f o g o f2 o g1)(f1(a1)) = (g1 o f o g o f2 o g1)(f1(a2))
. | | by Analyze 10
. | |
. | 10. g1 o f2 = Id
. | 11. f2 o g1 = Id
. | 12. Surj(A; B; f)
. | 13. a1 : A
. | 14. a2 : A
. | 15. f(a1) = f(a2)
. | ⊢   by RWH (HypC 11) 0 THEN Reduce 0
. | |
. | ⊢  g1(f(g(f1(a1)))) = g1(f(g(f1(a2))))  by Analyze 6
. | |
. | 6. g o f1 = Id
. | 7. f1 o g = Id
. | 8. n : ℕ
. | 9. f2 : ℕn→B
. | 10. g1 : B→ℕn
. | 11. g1 o f2 = Id
. | 12. f2 o g1 = Id
. | 13. Surj(A; B; f)
. | 14. a1 : A
. | 15. a2 : A
. | 16. f(a1) = f(a2)
. | ⊢   by RW (LowerC add_composeC) 0
. | |
. | ⊢  g1(f((g o f1)(a1))) = g1(f((g o f1)(a2)))
. | | by RWH (HypC 6) 0 THEN Reduce 0
. | |
. | ⊢  g1(f(a1)) = g1(f(a2))  by RWH (HypC 16) 0
. |
. .\
. . 15. g1(f1(a1)) = g1(f1(a2))
. . ⊢   by ApFunToHypEquands ‘z’ (f2(z)) B 15
. . |
. . 16. f2(g1(f1(a1))) = f2(g1(f1(a2)))
. . ⊢   by RWH add_composeC 16 THEN Analyze 10
. . |
. . 10. g1 o f2 = Id
. . 11. f2 o g1 = Id
. . 12. Surj(A; B; f)
. . 13. a1 : A
. . 14. a2 : A
. . 15. f(a1) = f(a2)
. . 16. g1(f1(a1)) = g1(f1(a2))
```

```
. . 17. (f2 o g1)(f1(a1)) = (f2 o g1)(f1(a2))
. . ⊢   by RWH (HypC 11) 17 THEN Reduce 17
. . |
. . 17. f1(a1) = f1(a2)
. . ⊢   by ApFunToHypEquands ‘z’ (g(z)) A 17
. . |
. . 18. g(f1(a1)) = g(f1(a2))
. . ⊢   by RWH add_composeC 18
. . |
. . 18. (g o f1)(a1) = (g o f1)(a2)
. . ⊢   by Analyze 6
. . |
. . 6. g o f1 = Id
. . 7. f1 o g = Id
. . 8. n : ℕ
. . 9. f2 : ℕn→B
. . 10. g1 : B→ℕn
. . 11. g1 o f2 = Id
. . 12. f2 o g1 = Id
. . 13. Surj(A; B; f)
. . 14. a1 : A
. . 15. a2 : A
. . 16. f(a1) = f(a2)
. . 17. g1(f1(a1)) = g1(f1(a2))
. . 18. f1(a1) = f1(a2)
. . 19. (g o f1)(a1) = (g o f1)(a2)
. . ⊢   by RWH (HypC 6) 19 THEN Reduce 19
END
```