```
comp_choice  THM

⊢  ∀E:(ℕ→ℕ→Prop).
|  (EquivRel x,y:ℕ. x E y) & (∀x,y:ℕ. Dec(x E y)) ⇒
|  (∃h:(ℕ→ℕ). (∀n,k:ℕ. (n E k) ⟺ h(n) = h(k)) & (∀n:ℕ. n E (h(n))))
| by UnivCD THEN Analyze 2
|
1. E : ℕ→ℕ→Prop
2. EquivRel x,y:ℕ. x E y
3. ∀x,y:ℕ. Dec(x E y)
⊢  ∃h:(ℕ→ℕ). (∀n,k:ℕ. (n E k) ⟺ h(n) = h(k)) & (∀n:ℕ. n E (h(n)))
| by FwdThru
|    Thm* ∀R:(T→T→Prop).
|          (∀x,y:T. Dec(x R y)) ⇒
|          (∃r:(T→T→𝔹). ∀x,y:T. (x r y) ⟺ (x R y))
|    {decide_imp_bool}
|    [3]
|
4. ∃r:(ℕ→ℕ→𝔹). ∀x,y:ℕ. (x r y) ⟺ (x E y)
⊢   by Analyze 4
|
4. r : ℕ→ℕ→𝔹
5. ∀x,y:ℕ. (x r y) ⟺ (x E y)
⊢   by Witness λz.Min{ x | xrz } THEN Reduce 0
|
⊢  (∀n,k:ℕ. (n E k) ⟺ Min{ x | xrn } = Min{ x | xrk })
|  & (∀n:ℕ. n E Min{ x | xrn })  by RWH (RevHypC 5) 0
|
⊢  (∀n,k:ℕ. (n r k) ⟺ Min{ x | xrn } = Min{ x | xrk })
|  & (∀n:ℕ. n r Min{ x | xrn })  by RWH (RevHypC 5) 2
|
2. ∀x,y:ℕ. Dec(x E y)
3. r : ℕ→ℕ→𝔹
4. ∀x,y:ℕ. (x r y) ⟺ (x E y)
5. EquivRel x,y:ℕ. x r y
⊢   by UnivCD
|
⊢   by Analyze 0
|
|\
| ⊢  ∀n,k:ℕ. (n r k) ⟺ Min{ x | xrn } = Min{ x | xrk }  by UnivCD
| |
| 6. n : ℕ
| 7. k : ℕ
| ⊢  (n r k) ⟺ Min{ x | xrn } = Min{ x | xrk }
|   by Inst
|      Thm* ∀E:(ℕ→ℕ→𝔹), n,k:ℕ.
|           (EquivRel x,y:ℕ. x E y) ⇒
|           ((n E k) ⟺ Min{ x | xEn } = Min{ x | xEk })
|      {min_el_iff}
|      [r;n;k]
|
.\
. ⊢  ∀n:ℕ. n r Min{ x | xrn }
.   by Analyze 0
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.       THEN
.       BackThru
.       Thm* ∀E:(ℕ→ℕ→𝔹), n:ℕ.
.            (EquivRel x,y:ℕ. x E y) ⇒ (n E Min{ x | xEn })
.       {min_el_sound}
END
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