```
decide_imp_bool  THM

⊢  ∀R:(T→T→Prop).
|  (∀x,y:T. Dec(x R y)) ⇒ (∃r:(T→T→𝔹). ∀x,y:T. (x r y) ⟺ (x R y))
| by UnivCD
|
1. T : Type
2. R : T→T→Prop
3. ∀x,y:T. Dec(x R y)
⊢  ∃r:(T→T→𝔹). ∀x,y:T. (x r y) ⟺ (x R y)
| by Inst
|    Thm* ∀Q:(A→B→Prop).
|         (∀x:A. ∃y:B. Q(x,y)) ⇒ (∃f:(A→B). ∀x:A. Q(x,f(x)))
|    {ax_choice}
|    [T;T→𝔹;λ₂x,y. ∀z:T. y(z) ⟺ (x R z)]
|
|\
| .....antecedent .....
| ⊢  ∀x:T. ∃y:(T→𝔹). ∀z:T. y(z) ⟺ (x R z)  by Analyze 0
| |
| 4. x : T
| ⊢  ∃y:(T→𝔹). ∀z:T. y(z) ⟺ (x R z)
| | by Inst
| |    Thm* ∀Q:(A→B→Prop).
| |         (∀x:A. ∃y:B. Q(x,y)) ⇒ (∃f:(A→B). ∀x:A. Q(x,f(x)))
| |    {ax_choice}
| |    [T;𝔹;λ₂z,y. y ⟺ (x R z)]
| |
| .....antecedent .....
| ⊢  ∀x@0:T. ∃y:𝔹. y ⟺ (x R x@0)  by Analyze 0
| |
| 5. x@0 : T
| ⊢  ∃y:𝔹. y ⟺ (x R x@0)  by Decide (x R x@0)
| |
| |\
| | 6. x R x@0
| | ⊢   by Witness true₂ THEN Reduce 0
| |
| .\
| . 6. ¬(x R x@0)
| . ⊢   by Witness false₂ THEN Reduce 0
|
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. 4. ∃f:(T→T→𝔹). ∀x,z:T. f(x,z) ⟺ (x R z)
. ⊢   by Unfold ‘infix_ap‘ 0
END
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