```
min_ar_alt  THM

⊢  ∀f:(ℕ→𝔹), n:ℕ, i:ℕ(n+1), j:ℕ(n-i+1).
|  f(n-i) = f(n) ⇒ MinAr(f;j+i;n) = MinAr(f;j;n-i)+i
| by UnivCD THEN NSubsetInd 4
|
|\
| .....basecase .....
| 1. f : ℕ→𝔹
| 2. n : ℕ
| 3. i : ℕ(n+1)
| 4. f(n-i) = f(n)
| ⊢  MinAr(f;0+i;n) = MinAr(f;0;n-i)+i
| | by Assert (0+i = i) THEN RWH (HypC 5) 0 THEN Thin 5
| |
| ⊢  MinAr(f;i;n) = MinAr(f;0;n-i)+i
| | by RW (AddrC [2] (RecUnfoldC `min_ar`)) 0
| |
| ⊢  if (f(n-i))=₂(f(n))→ i else MinAr(f;i-1;n) fi = MinAr(f;0;n-i)+i
|   by SplitOnConclITE
|
.\
. .....upcase .....
. 1. f : ℕ→𝔹
. 2. n : ℕ
. 3. i : ℕ(n+1)
. 4. f(n-i) = f(n)
. 5. j : ℤ
. 6. 0<j
. 7. j<n-i+1
. 8. MinAr(f;j-1+i;n) = MinAr(f;j-1;n-i)+i
. ⊢  MinAr(f;j+i;n) = MinAr(f;j;n-i)+i
. | by RW (AddrC [2] (RecUnfoldC `min_ar`)) 0 THEN SplitOnConclITE
. |
. |\
. | .....truecase .....
. | 9. f(n-(j+i)) = f(n)
. | ⊢  j+i = MinAr(f;j;n-i)+i  by RecCaseSplit `min_ar`
. | |
. | |\
. | | .....truecase .....
. | | 10. f(n-i-j) = f(n-i)
. | | ⊢  j+i = j+i  by Auto
. | |
. | .\
. | . .....falsecase .....
. | . 10. ¬f(n-i-j) = f(n-i)
. | . ⊢  j+i = MinAr(f;j-1;n-i)+i
. | . | by Assert (n-(j+i) = n-i-j) THEN RWH (HypC 11) 9 THEN Thin 11
. | . |
. | . 9. f(n-i-j) = f(n)
. | . 10. ¬f(n-i-j) = f(n-i)
. | . ⊢   by Analyze 10
. |
. .\
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. . .....falsecase .....
. . 9. ¬f(n-(j+i)) = f(n)
. . ⊢  MinAr(f;j+i-1;n) = MinAr(f;j;n-i)+i
. . | by Assert (j-1+i = j+i-1) THEN RWH (HypC 10) 8 THEN Thin 10
. . |
. . 8. MinAr(f;j+i-1;n) = MinAr(f;j-1;n-i)+i
. . 9. ¬f(n-(j+i)) = f(n)
. . ⊢
. . | by RW (AddrC [3] (RecUnfoldC `min_ar`)) 0 THEN SplitOnConclITE
. . |
. . |\
. . | .....truecase .....
. . | 10. f(n-i-j) = f(n-i)
. . | ⊢  MinAr(f;j+i-1;n) = j+i
. . |   by Assert (n-i-j = n-(j+i)) THEN RWH (HypC 11) 10 THEN Thin 11
. . |      THEN
. . |      Analyze 9
. . |
. . .\
. . . .....falsecase .....
. . . 10. ¬f(n-i-j) = f(n-i)
. . . ⊢  MinAr(f;j+i-1;n) = MinAr(f;j-1;n-i)+i  by Auto
END
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