```
min_el_iff  THM

⊢  ∀E:(ℕ→ℕ→𝔹), n,k:ℕ.
|  (EquivRel x,y:ℕ. x E y) ⇒
|  ((n E k) ⟺ Min{ x | xEn } = Min{ x | xEk })
| by UnivCD THEN Analyze 0
|
|\
| 1. E : ℕ→ℕ→𝔹
| 2. n : ℕ
| 3. k : ℕ
| 4. EquivRel x,y:ℕ. x E y
| ⊢  (n E k) ⇒ Min{ x | xEn } = Min{ x | xEk }  by Analyze 0
| |
| 5. n E k
| ⊢  Min{ x | xEn } = Min{ x | xEk }
|   by BackThru
|      Thm* ∀E:(ℕ→ℕ→𝔹), n,k:ℕ.
|           (EquivRel x,y:ℕ. x E y) ⇒
|           (n E k) ⇒ Min{ x | xEn } = Min{ x | xEk }
|      {min_el_unique}
|
.\
. 1. E : ℕ→ℕ→𝔹
. 2. n : ℕ
. 3. k : ℕ
. 4. EquivRel x,y:ℕ. x E y
. ⊢  n E k ⇐ (Min{ x | xEn } = Min{ x | xEk })  by Analyze 0
. |
. 5. Min{ x | xEn } = Min{ x | xEk }
. ⊢  n E k
. | by Inst
. |    Thm* ∀E:(ℕ→ℕ→𝔹), n:ℕ.
. |         (EquivRel x,y:ℕ. x E y) ⇒ (n E Min{ x | xEn })
. |    {min_el_sound}
. |    [E;n]
. |    THEN
. |    Inst
. |    Thm* ∀E:(ℕ→ℕ→𝔹), n:ℕ.
. |         (EquivRel x,y:ℕ. x E y) ⇒ (n E Min{ x | xEn })
. |    {min_el_sound}
. |    [E;k]
. |
. 6. n E Min{ x | xEn }
. 7. k E Min{ x | xEk }
. ⊢   by Analyze 4 THEN Analyze 5
. |
. 4. Refl(ℕ;x,y.x E y)
. 5. Sym x,y:ℕ. x E y
. 6. Trans x,y:ℕ. x E y
. 7. Min{ x | xEn } = Min{ x | xEk }
. 8. n E Min{ x | xEn }
. 9. k E Min{ x | xEk }
. ⊢   by RWH (HypC 7) 8
. |
```

```
. 8. n E Min{ x | xEk }
. 9. k E Min{ x | xEk }
. ⊢   by Unfold ‘sym‘ 5
. |
. 5. ∀a,b:ℕ. (a E b) ⇒ (b E a)
. 6. Trans x,y:ℕ. x E y
. 7. Min{ x | xEn } = Min{ x | xEk }
. 8. n E Min{ x | xEk }
. 9. k E Min{ x | xEk }
. ⊢   by FwdThru 5 [9]
. |
. 10. Min{ x | xEk } E k
. ⊢   by Unfold ‘trans‘ 6
. |
. 6. ∀a,b,c:ℕ. (a E b) ⇒ (b E c) ⇒ (a E c)
. 7. Min{ x | xEn } = Min{ x | xEk }
. 8. n E Min{ x | xEk }
. 9. k E Min{ x | xEk }
. 10. Min{ x | xEk } E k
. ⊢   by FwdThru 6 [8;10]
END
```