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min_el_unique  THM

⊢  ∀E:(ℕ→ℕ→𝔹), n,k:ℕ.
|  (EquivRel x,y:ℕ. x E y) ⇒
|  (n E k) ⇒ Min{ x | xEn } = Min{ x | xEk }  by UnivCD
|
1. E : ℕ→ℕ→𝔹
2. n : ℕ
3. k : ℕ
4. EquivRel x,y:ℕ. x E y
5. n E k
⊢  Min{ x | xEn } = Min{ x | xEk }  by Unfold ‘min_el‘ 0
|
⊢  MinArg(E(n) : {0..n+}) = MinArg(E(k) : {0..k+})
| by FwdThru
|    Thm* ∀E:(T→T→𝔹), s,t:T.
|         (EquivRel x,y:T. x E y) ⇒ (s E t) ⇒ E(s) = E(t)
|    {equiv_rel_fun}
|    [4;5]
|
6. E(n) = E(k)
⊢   by RWH (HypC 6) 0
|
⊢  MinArg(E(k) : {0..n+}) = MinArg(E(k) : {0..k+})
| by BackThru
|    Thm* ∀f:(ℕ→𝔹), n,k:ℕ.
|         f(n) = f(k) ⇒ MinArg(f : {0..n+}) = MinArg(f : {0..k+}) ∈ ℕ
|    {min_arg_unique}
|
⊢  E(k,n) = E(k,k)  by Analyze 4 THEN Analyze 5 THEN Unfold ‘sym‘ 5
|
4. Refl(ℕ;x,y.x E y)
5. ∀a,b:ℕ. (a E b) ⇒ (b E a)
6. Trans x,y:ℕ. x E y
7. n E k
8. E(n) = E(k)
⊢   by FwdThru 5 [7]
|
9. k E n
⊢   by FwdThru Thm* ∀b:𝔹. b ⇒ b = true₂    {assert_elim} [9]
|
10. (k E n) = true₂
⊢   by Unfold ‘refl‘ 4
|
4. ∀a:ℕ. a E a
5. ∀a,b:ℕ. (a E b) ⇒ (b E a)
6. Trans x,y:ℕ. x E y
7. n E k
8. E(n) = E(k)
9. k E n
10. (k E n) = true₂
⊢   by Witness4 k
|
4. ∀a,b:ℕ. (a E b) ⇒ (b E a)
5. Trans x,y:ℕ. x E y
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6. n E k
7. E(n) = E(k)
8. k E n
9. (k E n) = true₂
10. k E k
⊢   by FwdThru Thm* ∀b:𝔹. b ⇒ b = true₂    {assert_elim} [10]
|
11. (k E k) = true₂
⊢   by Fold ‘infix_ap‘ 0
END
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