```
exists_functionality_wrt_iff  THM

⊢  ∀P,Q:(S→Prop).
|  S = T ⇒ (∀x:S. P(x) ⟺ Q(x)) ⇒ ((∃x:S. P(x)) ⟺ (∃y:T. Q(y)))
| by GenUnivCD
|
|\
| 1. S : Type
| 2. T : Type
| 3. P : S→Prop
| 4. Q : S→Prop
| 5. S = T
| 6. ∀x:S. P(x) ⟺ Q(x)
| 7. ∃x:S. P(x)
| ⊢  ∃y:T. Q(y)  by (Analyze 7 THEN Witness x) THEN HypBackchain
|
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. 1. S : Type
. 2. T : Type
. 3. P : S→Prop
. 4. Q : S→Prop
. 5. S = T
. 6. ∀x:S. P(x) ⟺ Q(x)
. 7. ∃y:T. Q(y)
. ⊢  ∃x:S. P(x)  by (Analyze 7 THEN Witness y) THEN HypBackchain
END
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