```
decidable_iff_exists_bool_function  THM

⊢  ∀P:(T→Prop).
|  (∀x:T. Dec(P(x))) ⇒ (∃f_p:(T→𝔹). ∀x:T. P(x) ⟺ f_p(x))
| by UnivCD
|
1. T : Type
2. P : T→Prop
3. ∀x:T. Dec(P(x))
⊢  ∃f_p:(T→𝔹). ∀x:T. P(x) ⟺ f_p(x)
| by Repeat (Unfolds [‘decidable‘;‘or‘;‘not‘;‘all‘;‘implies‘] 3)
|    THEN
|    RenameVar ‘g’ 3
|
3. x:T→P(x)+(P(x)→False)
⊢  ∃f_p:(T→𝔹). ∀x:T. P(x) ⟺ f_p(x)
| by Witness λx.InjCase(g(x) ; true₂; false₂) THEN Analyze 0
|    THEN
|    Reduce 0
|
4. x : T
⊢  P(x) ⟺ InjCase(g(x) ; true₂; false₂)  by Witness3 x
|
5. y : P(x)+(P(x)→False)
6. y = g(x)
⊢  P(x) ⟺ InjCase(g(x) ; true₂; false₂)
| by RevHypSubst 6 0 THEN Analyze 5 THEN Reduce 0
|
5. y1 : P(x)→False
6. inr(y1) = g(x)
7. P(x)
⊢  False  by Analyze 5
END
```