```
discrete_equality_is_equivalence  THM

⊢  ∀EQ:{T=₂}. EQ ∈ {T≡₂}  by UnivCD
|
1. T : Type
2. EQ : {T=₂}
⊢  EQ ∈ {T≡₂}
| by Unfold ‘equivalence‘ 0 THEN DiscreteEq 2 THEN AbSetMemEqTypeCD
|
|\
| 3. EQ ∈ T→T→𝔹
| 4. ∀x,y:T. EQ(x,y) ⟺ x = y
| ⊢  EQ ∈ T→T→𝔹  by Auto
|
|\
| 3. EQ ∈ T→T→𝔹
| 4. ∀x,y:T. EQ(x,y) ⟺ x = y
| 5. x : T
| ⊢  EQ(x,x)
|   by Witness4 x THEN With x (Analyze -1) THEN HypBackchain
|
|\
| 3. EQ ∈ T→T→𝔹
| 4. ∀x,y:T. EQ(x,y) ⟺ x = y
| 5. x : T
| 6. y : T
| 7. EQ(x,y)
| ⊢  EQ(y,x)
| | by CopyToEnd 4 THEN With y (Analyze -1) THEN With x (Analyze -1)
| |    THEN
| |    CopyToEnd 4
| |    THEN
| |    With x (Analyze -1)
| |    THEN
| |    With y (Analyze -1)
| |
| 8. EQ(y,x) ⟺ y = x
| 9. EQ(x,y) ⟺ x = y
| ⊢  EQ(y,x)  by HypBackchain THEN HypBackchain
|
|\
| 3. EQ ∈ T→T→𝔹
| 4. ∀x,y:T. EQ(x,y) ⟺ x = y
| 5. x : T
| 6. y : T
| 7. EQ(y,x)
| ⊢  EQ(x,y)
| | by CopyToEnd 4 THEN With y (Analyze -1) THEN With x (Analyze -1)
| |    THEN
| |    CopyToEnd 4
| |    THEN
| |    With x (Analyze -1)
| |    THEN
| |    With y (Analyze -1)
| |
```

```
| 8. EQ(y,x) ⟺ y = x
| 9. EQ(x,y) ⟺ x = y
| ⊢  EQ(x,y)  by Assert (x = y ⟺ y = x)
| |
| |\
| | .....assertion .....
| | ⊢  x = y ⟺ y = x  by Auto
| |
| .\
| . 10. x = y ⟺ y = x
| . ⊢  EQ(x,y)  by Repeat HypBackchain
|
.\
. 3. EQ ∈ T→T→𝔹
. 4. ∀x,y:T. EQ(x,y) ⟺ x = y
. 5. x : T
. 6. y : T
. 7. z : T
. 8. EQ(x,y)
. 9. EQ(y,z)
. ⊢  EQ(x,z)
. | by CopyToEnd 4 THEN With x (Analyze -1) THEN With y (Analyze -1)
. |    THEN
. |    CopyToEnd 4
. |    THEN
. |    With y (Analyze -1)
. |    THEN
. |    With z (Analyze -1)
. |    THEN
. |    CopyToEnd 4
. |    THEN
. |    With x (Analyze -1)
. |    THEN
. |    With z (Analyze -1)
. |
. 10. EQ(x,y) ⟺ x = y
. 11. EQ(y,z) ⟺ y = z
. 12. EQ(x,z) ⟺ x = z
. ⊢  EQ(x,z)  by FwdThru 10 [8] THEN FwdThru 11 [9]
. |
. 13. x = y
. 14. y = z
. ⊢  EQ(x,z)  by RevHypSubst 13 14 THEN HypBackchain
END
```