```
equivalence_properties  THM

⊢  ∀f:{T≡₂}.
|  (∀x,y,z:T. f(x,y) ⇒ f(y,z) ⇒ f(x,z))
|  & (∀x,y:T. f(x,y) ⟺ f(y,x))
|  & (∀x:T. f(x,x))
|  & f ∈ T→T→𝔹  by ProvePropertiesLemma
|
|\
| 1. T : Type
| 2. f : T→T→𝔹
| 3. ∀x:T. f(x,x)
| 4. ∀x,y:T. f(x,y) ⟺ f(y,x)
| 5. ∀x,y,z:T. f(x,y) ⇒ f(y,z) ⇒ f(x,z)
| 6. x : T
| 7. y : T
| 8. z : T
| 9. f(x,y)
| 10. f(y,z)
| ⊢  f(x,z)
| | by Witness5 x THEN With y (Analyze -1) THEN With z (Analyze -1)
| |
| 5. x : T
| 6. y : T
| 7. z : T
| 8. f(x,y)
| 9. f(y,z)
| 10. f(x,y) ⇒ f(y,z) ⇒ f(x,z)
| ⊢  f(x,z)  by HypBackchain
|
|\
| 1. T : Type
| 2. f : T→T→𝔹
| 3. ∀x:T. f(x,x)
| 4. ∀x,y:T. f(x,y) ⟺ f(y,x)
| 5. ∀x,y,z:T. f(x,y) ⇒ f(y,z) ⇒ f(x,z)
| 6. x : T
| 7. y : T
| 8. f(x,y)
| ⊢  f(y,x)  by Witness4 x THEN With y (Analyze -1) THEN HypBackchain
|
|\
| 1. T : Type
| 2. f : T→T→𝔹
| 3. ∀x:T. f(x,x)
| 4. ∀x,y:T. f(x,y) ⟺ f(y,x)
| 5. ∀x,y,z:T. f(x,y) ⇒ f(y,z) ⇒ f(x,z)
| 6. x : T
| 7. y : T
| 8. f(y,x)
| ⊢  f(x,y)  by Witness4 x THEN With y (Analyze -1) THEN HypBackchain
|
.\
. 1. T : Type
. 2. f : T→T→𝔹
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. 3. ∀x:T. f(x,x)
. 4. ∀x,y:T. f(x,y) ⟺ f(y,x)
. 5. ∀x,y,z:T. f(x,y) ⇒ f(y,z) ⇒ f(x,z)
. 6. x : T
. ⊢  f(x,x)  by Witness3 x THEN Trivial
END
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