```
zero_rank_all_vars  THM

⊢  ∀L:Sequent List, eqS:{Sequent=2}, eqF:{Formula=2}.
|  ∀s∈L.(ρ(s) = 0) ⇒
|  (∀hyp,concl:Formula List.
|   <hyp,concl>(∈eqS) L ⇒
|   (∀z:Formula. z(∈eqF) hyp ∨ z(∈eqF) concl ⇒ (∃v:Var. z = ⌜v⌝)))
| by UnivCD
|
1. L : Sequent List
2. eqS : {Sequent=2}
3. eqF : {Formula=2}
4. ∀s∈L.(ρ(s) = 0)
5. hyp : Formula List
6. concl : Formula List
7. <hyp,concl>(∈eqS) L
8. z : Formula
9. z(∈eqF) hyp ∨ z(∈eqF) concl
⊢  ∃v:Var. z = ⌜v⌝  by Analyze -1
|
|\
| 9. z(∈eqF) hyp
| ⊢
| | by Using
| |    [‘T’,Sequent;‘eq’,eqS;‘P’,(λ2s. ρ(s) = 0);‘z’,<hyp,concl>]
| |    (FwdThru
| |     Thm* ∀eq:{T=2}, P:(T→Prop), L:T List.
| |          ∀x∈L.P(x) ⟺ (∀z:T. z(∈eq) L ⇒ P(z))
| |     {list_all_is_member_lemma}
| |     [4])
| |
| 10. ρ(<hyp,concl>) = 0
| ⊢   by AbReduce -1
| |
| 10. ρ(hyp)+ρ(concl) = 0
| ⊢
| | by FwdThru
| |    Thm* ∀eq:{T=2}, L:T List, x:T.
| |         x(∈eq) L ⇒ (∃M,N:T List. L = (M @ (x.N)))
| |    {is_member_append_lemma}
| |    [-2]
| |    THEN
| |    Repeat (Analyze -1)
| |
| 11. M : Formula List
| 12. N : Formula List
| 13. hyp = (M @ (z.N))
| ⊢   by MoveToEnd -4 THEN HypSubst -2 -1 THEN AbReduce -1
| |
| 10. M : Formula List
| 11. N : Formula List
| 12. hyp = (M @ (z.N))
| 13. ρ(M @ (z.N))+ρ(concl) = 0
| ⊢   by Rewrite (HigherC list_rank_append_conv) -1 THEN AbReduce -1
| |
```

```
| 13. ρ(M)+ρ(z)+ρ(N)+ρ(concl) = 0
| ⊢   by Assert (ρ(z) = 0)
| |
| |\
| | .....assertion .....
| | ⊢  ρ(z) = 0  by MoveToConcl -1 THEN GenConclOnAps
| |
| .\
| . 14. ρ(z) = 0
| . ⊢   by MoveToConcl -1 THEN TryOnAllHyps Thin
| . |
| . 1. z : Formula
| . ⊢  ρ(z) = 0 ⇒ (∃v:Var. z = ⌈v⌉)
| . | by FormulaCases 1 THEN AbReduce 0
| . |
| . |\
| . | 2. x : Var
| . | ⊢  0 = 0 ⇒ (∃v:Var. ⌈x⌉ = ⌈v⌉)  by Analyze 0
| . | |
| . | 3. 0 = 0
| . | ⊢  ∃v:Var. ⌈x⌉ = ⌈v⌉  by With x (Analyze 0)
| . |
| . |\
| . | 2. x : Formula
| . | ⊢  ρ(x)+1 = 0 ⇒ (∃v:Var. ⌈◇⌉x = ⌈v⌉)  by GenConclOnAps
| . |
| . |\
| . | 2. x1 : Formula
| . | 3. x2 : Formula
| . | ⊢  ρ(x1)+ρ(x2)+1 = 0 ⇒ (∃v:Var. x1⌈∧⌉x2 = ⌈v⌉)
| . |   by GenConclOnAps
| . |
| . |\
| . | 2. x1 : Formula
| . | 3. x2 : Formula
| . | ⊢  ρ(x1)+ρ(x2)+1 = 0 ⇒ (∃v:Var. x1⌈∨⌉x2 = ⌈v⌉)
| . |   by GenConclOnAps
| . |
| . .\
| . . 2. y1 : Formula
| . . 3. y2 : Formula
| . . ⊢  ρ(y1)+ρ(y2)+1 = 0 ⇒ (∃v:Var. y1⌈⇒⌉y2 = ⌈v⌉)
| . .   by GenConclOnAps
|
.\
. 9. z(∈eqF) concl
. ⊢

. | by Using
. |    [‘T’,Sequent;‘eq’,eqS;‘P’,(λ2s. ρ(s) = 0);‘z’,<hyp,concl>]
. |    (FwdThru
. |     Thm* ∀eq:{T=2}, P:(T→Prop), L:T List.
. |          ∀x∈L.P(x) ⟺ (∀z:T. z(∈eq) L ⇒ P(z))
. |     {list_all_is_member_lemma}
. |     [4])
. |
```

```
. 10. ρ(<hyp,concl>) = 0
. ⊢   by AbReduce -1
. |
. 10. ρ(hyp)+ρ(concl) = 0
. ⊢
. | by FwdThru
. |    Thm* ∀eq:{T=₂}, L:T List, x:T.
. |         x(∈eq) L ⇒ (∃M,N:T List. L = (M @ (x.N)))
. |    {is_member_append_lemma}
. |    [-2]
. |    THEN
. |    Repeat (Analyze -1)
. |
. 11. M : Formula List
. 12. N : Formula List
. 13. concl = (M @ (z.N))
. ⊢   by MoveToEnd -4 THEN HypSubst -2 -1 THEN AbReduce -1
. |
. 10. M : Formula List
. 11. N : Formula List
. 12. concl = (M @ (z.N))
. 13. ρ(hyp)+ρ(M @ (z.N)) = 0
. ⊢   by Rewrite (HigherC list_rank_append_conv) -1 THEN AbReduce -1
. |
. 13. ρ(hyp)+ρ(M)+ρ(z)+ρ(N) = 0
. ⊢   by Assert (ρ(z) = 0)
. |
. |\
. | .....assertion .....
. | ⊢  ρ(z) = 0  by MoveToConcl -1 THEN GenConclOnAps
. |
. .\
. . 14. ρ(z) = 0
. . ⊢   by MoveToConcl -1 THEN TryOnAllHyps Thin
. . |
. . 1. z : Formula
. . ⊢  ρ(z) = 0 ⇒ (∃v:Var. z = ⌈v⌉)
. . | by FormulaCases 1 THEN AbReduce 0
. . |
. . |\
. . | 2. x : Var
. . | ⊢  0 = 0 ⇒ (∃v:Var. ⌈x⌉ = ⌈v⌉)  by Analyze 0
. . | |
. . | 3. 0 = 0
. . | ⊢  ∃v:Var. ⌈x⌉ = ⌈v⌉  by With x (Analyze 0)
. . |
. . |\
. . | 2. x : Formula
. . | ⊢  ρ(x)+1 = 0 ⇒ (∃v:Var. ⌈◇⌉x = ⌈v⌉)  by GenConclOnAps
. . |
. . |\
. . | 2. x1 : Formula
. . | 3. x2 : Formula
. . | ⊢  ρ(x1)+ρ(x2)+1 = 0 ⇒ (∃v:Var. x1⌈∧⌉x2 = ⌈v⌉)
. . |   by GenConclOnAps
```

```
. . |
. . |\
. . | 2. x1 : Formula
. . | 3. x2 : Formula
. . | ⊢  ρ(x1)+ρ(x2)+1 = 0 ⇒ (∃v:Var. x1⌈∨⌉x2 = ⌈v⌉)
. . |   by GenConclOnAps
. . |
. . .\
. . . 2. y1 : Formula
. . . 3. y2 : Formula
. . . ⊢  ρ(y1)+ρ(y2)+1 = 0 ⇒ (∃v:Var. y1⌈⇒⌉y2 = ⌈v⌉)
. . .   by GenConclOnAps
END
```