```
discrete__union  THM

⊢  ∀T1:Type{i}, T2:Type{j}.
|  Discrete{T1} ⇒ Discrete{T2} ⇒ Discrete{(T1+T2)}  by UnivCD
|
1. T1 : Type
2. T2 : Type
3. Discrete{T1}
4. Discrete{T2}
⊢  Discrete{(T1+T2)}
| by OnHyps [0;3;4] (λi.UnfoldTopAb i) THEN UnivCD
|
3. ∀x,y:T1. Dec(x = y)
4. ∀x,y:T2. Dec(x = y)
5. x : T1+T2
6. y : T1+T2
⊢  Dec(x = y)  by UnfoldTopAb 0 THEN Analyze 6 THEN Analyze 5
|
|\
| 5. x2 : T1
| 6. x1 : T1
| ⊢  inl(x2) = inl(x1) ∨ ¬inl(x2) = inl(x1)
| | by With x2 (Analyze 3) THEN With x1 (Analyze -1) THEN Analyze -1
| |
| |\
| | 3. ∀x,y:T2. Dec(x = y)
| | 4. x2 : T1
| | 5. x1 : T1
| | 6. x2 = x1
| | ⊢  inl(x2) = inl(x1) ∨ ¬inl(x2) = inl(x1)  by DNth 1 0
| | |
| | ⊢  inl(x2) = inl(x1)  by MemberEqCD
| | |
| | ⊢  x2 = x1  by Trivial
| |
| .\
| . 3. ∀x,y:T2. Dec(x = y)
| . 4. x2 : T1
| . 5. x1 : T1
| . 6. ¬x2 = x1
| . ⊢  inl(x2) = inl(x1) ∨ ¬inl(x2) = inl(x1)  by DNth 2 0
| . |
| . ⊢  ¬inl(x2) = inl(x1)  by UnfoldTopAb 0 THEN Analyze 0
| . |
| . 7. inl(x2) = inl(x1)
| . ⊢  False
| . | by ApFunToHypEquands ‘z’ InjCase(z; z. z; . x1) T1 7
| . |    THEN
| . |    Reduce -1
| . |
| . 8. x2 = x1
| . ⊢  False  by UnfoldTopAb 6 THEN FwdThru 6 [8] THEN Trivial
|
|\
| 5. y : T2
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| 6. x1 : T1
| ⊢  inr(y) = inl(x1) ∨ ¬inr(y) = inl(x1)  by DNth 2 0
| |
| ⊢  ¬inr(y) = inl(x1)  by Analyze 0
| |
| 7. inr(y) = inl(x1)
| ⊢  False
| | by ApFunToHypEquands ‘z’ InjCase[z]( 0 ; 1) ℤ 7 THEN Reduce -1
| |
| 8. 1 = 0
| ⊢  False  by Auto
|
|\
| 5. x1 : T1
| 6. y1 : T2
| ⊢  inl(x1) = inr(y1) ∨ ¬inl(x1) = inr(y1)  by DNth 2 0
| |
| ⊢  ¬inl(x1) = inr(y1)  by Analyze 0
| |
| 7. inl(x1) = inr(y1)
| ⊢  False
| | by ApFunToHypEquands ‘z’ InjCase[z]( 0 ; 1) ℤ 7 THEN Reduce -1
| |
| 8. 0 = 1
| ⊢  False  by Auto
|
.\
. 5. y : T2
. 6. y1 : T2
. ⊢  inr(y) = inr(y1) ∨ ¬inr(y) = inr(y1)
. | by With y (Analyze 4) THEN With y1 (Analyze -1) THEN Analyze -1
. |
. |\
. | 4. y : T2
. | 5. y1 : T2
. | 6. y = y1
. | ⊢  inr(y) = inr(y1) ∨ ¬inr(y) = inr(y1)  by DNth 1 0
. | |
. | ⊢  inr(y) = inr(y1)  by MemberEqCD
. | |
. | ⊢  y = y1  by Trivial
. |
. .\
. . 4. y : T2
. . 5. y1 : T2
. . 6. ¬y = y1
. . ⊢  inr(y) = inr(y1) ∨ ¬inr(y) = inr(y1)  by DNth 2 0
. . |
. . ⊢  ¬inr(y) = inr(y1)  by UnfoldTopAb 0 THEN Analyze 0
. . |
. . 7. inr(y) = inr(y1)
. . ⊢  False
. . | by ApFunToHypEquands ‘z’ InjCase[z]( y1 ; z) T2 7
. . |    THEN
. . |    Reduce -1
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. . |
. . 8. y = y1
. . ⊢  False  by UnfoldTopAb 6 THEN FwdThru 6 [8] THEN Trivial
END
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