discrete_equality_unique THM

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⊢  ∀f1,f2:{T=2}. f1 = f2 ∈ T→T→𝔹  by UnivCD
|
1. T : Type
2. f1 : {T=2}
3. f2 : {T=2}
⊢  f1 = f2  by Analyze 3 THEN Analyze 2
|
2. f1 : T→T→𝔹
3. (∀x,y:T. f1(x,y) ⟺ x = y) & f1 ∈ T→T→𝔹
4. f2 : T→T→𝔹
5. (∀x,y:T. f2(x,y) ⟺ x = y) & f2 ∈ T→T→𝔹
⊢  f1 = f2  by ExtWith [‘v’] [T→T→𝔹]
|
3. ∀x,y:T. f1(x,y) ⟺ x = y
4. f1 ∈ T→T→𝔹
5. f2 : T→T→𝔹
6. ∀x,y:T. f2(x,y) ⟺ x = y
7. f2 ∈ T→T→𝔹
8. v : T
⊢  f1(v) = f2(v)  by ExtWith [‘w’] [T→𝔹]
|
9. w : T
⊢  f1(v,w) = f2(v,w)
| by GenConcl (f1(v,w) = b1) THEN GenConcl (f2(v,w) = b2)
|
10. b1 : 𝔹
11. f1(v,w) = b1
12. b2 : 𝔹
13. f2(v,w) = b2
⊢  b1 = b2  by Witness3 v THEN With w (Analyze -1)
|
3. f1 ∈ T→T→𝔹
4. f2 : T→T→𝔹
5. ∀x,y:T. f2(x,y) ⟺ x = y
6. f2 ∈ T→T→𝔹
7. v : T
8. w : T
9. b1 : 𝔹
10. f1(v,w) = b1
11. b2 : 𝔹
12. f2(v,w) = b2
13. f1(v,w) ⟺ v = w
⊢  b1 = b2  by Witness5 v THEN With w (Analyze -1)
|
5. f2 ∈ T→T→𝔹
6. v : T
7. w : T
8. b1 : 𝔹
9. f1(v,w) = b1
10. b2 : 𝔹
11. f2(v,w) = b2
12. f1(v,w) ⟺ v = w
13. f2(v,w) ⟺ v = w
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⊢  b1 = b2  by HypSubst -3 -1
|
13. b2 ⟺ v = w
⊢  b1 = b2  by HypSubst -5 -2
|
12. b1 ⟺ v = w
13. b2 ⟺ v = w
⊢  b1 = b2
| by OnHyps [8;9] BoolInd THEN TryOnAllHyps Reduce THEN Reduce 0
|
|\
| 8. f1(v,w) = true_2
| 9. f2(v,w) = false_2
| 10. True ⇒ v = w
| 11. True ⇐ (v = w)
| 12. False ⇒ v = w
| 13. False ⇐ (v = w)
| ⊢  true_2 = false_2  by Analyze 10
| |
| 10. True ⇐ (v = w)
| 11. False ⇒ v = w
| 12. False ⇐ (v = w)
| 13. v = w
| ⊢  true_2 = false_2  by FwdThru -2 [-1]
|
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. 8. f1(v,w) = false_2
. 9. f2(v,w) = true_2
. 10. False ⇒ v = w
. 11. False ⇐ (v = w)
. 12. True ⇒ v = w
. 13. True ⇐ (v = w)
. ⊢  false_2 = true_2  by Analyze 12
. |
. 12. True ⇐ (v = w)
. 13. v = w
. ⊢  false_2 = true_2  by FwdThru -3 [-1]
END
```