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discrete_implies_bool_equality  THM

⊢  ∀T:Type. Discrete{T} ⇒ (∃f:(T→T→𝔹). ∀x,y:T. f(x,y) ⟺ x = y)
| by UnivCD
|
1. T : Type
2. Discrete{T}
⊢  ∃f:(T→T→𝔹). ∀x,y:T. f(x,y) ⟺ x = y  by RenameVar ‘g’ 2
|
2. Discrete{T}
⊢  ∃f:(T→T→𝔹). ∀x,y:T. f(x,y) ⟺ x = y
| by Repeat (Unfolds [‘discrete‘;‘decidable‘;‘or‘] 2)
|
2. ∀x,y:T. (x = y)+(¬x = y)
⊢  ∃f:(T→T→𝔹). ∀x,y:T. f(x,y) ⟺ x = y
| by With (λx,y. InjCase(g(x,y) ; true₂; false₂)) (Analyze 0)
|
|\
| 3. x : T
| 4. y : T
| ⊢  InjCase(g(x,y) ; true₂; false₂) ∈ 𝔹  by Reduce 0 THEN UnivCD
| |
| ⊢  InjCase(g(x,y) ; true₂; false₂) ∈ 𝔹
| | by Witness2 x THEN Witness5 y
| |
| 2. g : x:T→∀y:T. (x = y)+(¬x = y)
| 3. x : T
| 4. y : T
| 5. y1 : y:T→(x = y)+(¬x = y)
| 6. y1 = g(x) ∈ (∀y:T. (x = y)+(¬x = y))
| 7. y2 : (x = y)+(¬x = y)
| 8. y2 = y1(y)
| ⊢  InjCase(g(x,y) ; true₂; false₂) ∈ 𝔹
| | by RevHypSubst 6 0 THEN RevHypSubst 8 0
| |
| ⊢  InjCase(y2 ; true₂; false₂) ∈ 𝔹  by Analyze 7 THEN Reduce 0
|
|\
| ⊢  ∀x,y:T. (λx,y. InjCase(g(x,y) ; true₂; false₂))(x,y) ⟺ x = y
| | by Analyze 0 THEN Analyze 0
| |
| 3. x : T
| 4. y : T
| ⊢  (λx,y. InjCase(g(x,y) ; true₂; false₂))(x,y) ⟺ x = y
| | by Reduce 0 THEN Analyze 0 THEN Analyze 0
| |
| |\
| | 5. InjCase(g(x,y) ; true₂; false₂)
| | ⊢  x = y  by Witness2 x
| | |
| | 2. g : x:T→∀y:T. (x = y)+(¬x = y)
| | 3. x : T
| | 4. y : T
| | 5. InjCase(g(x,y) ; true₂; false₂)
| | 6. y1 : ∀y:T. (x = y)+(¬x = y)
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| | 7. y1 = g(x)
| | ⊢  x = y  by MoveToEnd 5 THEN RevHypSubst 6 -1
| | |
| | 5. y1 : ∀y:T. (x = y)+(¬x = y)
| | 6. y1 = g(x)
| | 7. InjCase(y1(y) ; true₂; false₂)
| | ⊢  x = y  by Witness5 y
| | |
| | 5. y1 : y:T→(x = y)+(¬x = y)
| | 6. y1 = g(x) ∈ (∀y:T. (x = y)+(¬x = y))
| | 7. InjCase(y1(y) ; true₂; false₂)
| | 8. y2 : (x = y)+(¬x = y)
| | 9. y2 = y1(y)
| | ⊢  x = y  by MoveToEnd 7 THEN RevHypSubst 8 -1
| | |
| | 7. y2 : (x = y)+(¬x = y)
| | 8. y2 = y1(y)
| | 9. InjCase(y2 ; true₂; false₂)
| | ⊢  x = y  by Analyze 7 THENL [Trivial;Id]
| | |
| | 7. y3 : ¬x = y
| | 8. inr(y3) = y1(y)
| | 9. InjCase(inr(y3) ; true₂; false₂)
| | ⊢  x = y  by Reduce 9
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| .\
| . 5. x = y
| . ⊢  InjCase(g(x,y) ; true₂; false₂)  by HypSubst 5 0
| . |
| . ⊢  InjCase(g(y,y) ; true₂; false₂)
| . | by Unfold `all` 2 THEN Witness2 y
| . |
| . 2. g : x,y:T→(x = y)+(¬x = y)
| . 3. x : T
| . 4. y : T
| . 5. x = y
| . 6. y1 : y@0:T→(y = y@0)+(¬y = y@0)
| . 7. y1 = g(y)
| . ⊢  InjCase(g(y,y) ; true₂; false₂)  by Witness6 y
| . |
| . 8. y2 : (y = y)+(¬y = y)
| . 9. y2 = y1(y)
| . ⊢  InjCase(g(y,y) ; true₂; false₂)
| . | by RevHypSubst 7 0 THEN RevHypSubst 9 0
| . |
| . ⊢  InjCase(y2 ; true₂; false₂)  by Analyze 8 THEN Reduce 0
| . |

| . |\
| . | 8. x1 : y = y
| . | 9. inl(x1) = y1(y)
| . | ⊢  True  by Trivial
| . |
| . .\
| . . 8. y3 : ¬y = y
| . . 9. inr(y3) = y1(y)
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| . . ⊢  False
| . . | by Unfold ‘not‘ 8 THEN Assert (y = y) THEN RenameVar ‘z’ -1
| . . |
| . . 8. y3 : y = y ⇒ False
| . . 9. inr(y3) = y1(y)
| . . 10. y = y
| . . ⊢  False  by Witness8 z THEN Trivial
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. 3. f : T→T→𝔹
. 4. x : T
. 5. y : T
. ⊢  f(x,y) ∈ 𝔹  by Auto
END
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