```
discrete_implies_discrete_equality  THM

⊢  Discrete{T} ⇒ (∃f:{T=₂}. True)  by Analyze 0
|
1. T : Type
⊢  Discrete{T} ⇒ (∃f:{T=₂}. True)  by Analyze 0
|
2. Discrete{T}
⊢  ∃f:{T=₂}. True
| by RenameVar ‘g’ 2
|    THEN
|    Repeat (Unfolds [‘discrete‘;‘decidable‘;‘or‘] 2)
|
2. ∀x,y:T. (x = y)+(¬x = y)
⊢  ∃f:{T=₂}. True
| by Witness λx,y. InjCase(g(x,y) ; true₂; false₂)
|    THENL
|    [Id;Auto;Auto]
|
.....wf .....
⊢  (λx,y. InjCase(g(x,y) ; true₂; false₂)) ∈ {T=₂}
| by Unfold ‘discrete_equality‘ 0 THEN AbSetMemEqTypeCD
|
|\
| ⊢  (λx,y. InjCase(g(x,y) ; true₂; false₂)) ∈ T→T→𝔹
|   by MemberEqCD THENL [MemberEqCD;Auto]
|
|\
| .....set predicate .....
| ⊢  ∀x,y:T. (λx,y. InjCase(g(x,y) ; true₂; false₂))(x,y) ⟺ x = y
| | by UnivCD
| |
| 3. x : T
| 4. y : T
| ⊢  (λx,y. InjCase(g(x,y) ; true₂; false₂))(x,y) ⟺ x = y
| | by Reduce 0
| |
| ⊢  InjCase(g(x,y) ; true₂; false₂) ⟺ x = y
| | by Witness2 x THEN RevHypSubst -1 0
| |
| 2. g : x:T→∀y:T. (x = y)+(¬x = y)
| 3. x : T
| 4. y : T
| 5. y1 : ∀y:T. (x = y)+(¬x = y)
| 6. y1 = g(x)
| ⊢  InjCase(y1(y) ; true₂; false₂) ⟺ x = y
| | by Witness5 y THEN RevHypSubst -1 0
| |
| 5. y1 : y:T→(x = y)+(¬x = y)
| 6. y1 = g(x)
| 7. y2 : (x = y)+(¬x = y)
| 8. y2 = y1(y)
| ⊢  InjCase(y2 ; true₂; false₂) ⟺ x = y  by Analyze 7 THEN Reduce 0
| |
| 7. y3 : ¬x = y
```

```
| 8. inr(y3) = y1(y)
| 9. x = y
| ⊢  False  by Analyze -3 THEN Trivial
|
.\
. .....wf .....
. 3. eq : T→T→𝔹
. ⊢  (∀x,y:T. eq(x,y) ⟺ x = y) ∈ Type  by Auto
END
```