```
absurd_nonfin_imp_unb_inf  THM

⊢  (∀A:Type. ¬Finite(A) ⇒ Unbounded(A))
|  ⇒
|  (∀D:Type. ¬¬(∃∈D) ⇒ (∃∈D))  by 2 Times Analyze THEN Witness1: ℕ×D
|
1. D : Type
2. ¬Finite(ℕ×D) ⇒ Unbounded(ℕ×D)
⊢  ¬¬(∃∈D) ⇒ (∃∈D)  by Guarding (∃∈D) SimilarTo: Hyp:-1
|
|\
| 2. Finite(ℕ×D)
| ⊢  ¬(∃∈D)  by Analyze-1
| |
| 2. n : ℕ
| 3. (ℕ×D) ∼ ℕn
| ⊢  ¬(∃∈D)
| | by SimilarTo ¬Infinite(ℕn)
| |    THENA
| |    BackThru: Thm*  ∀k:ℕ. ¬Infinite(ℕk)    {nsub_not_infinite}
| |
| 4. ∃∈D
| ⊢  Infinite(ℕn)
| | by FwdThru:
| |    Thm*  (A ∼ B) ⇒ Infinite(A) ⇒ Infinite(B)
| |    {ooc_preserves_infiniteness}
| |    on [ (ℕ×D) ∼ ℕn ]
| |
| .....antecedent .....
| ⊢  Infinite(ℕ×D)  by New:d Analyze4 THEN Witness: λi.<i,d>
| |
| 4. d : D
| ⊢  Inj(ℕ; ℕ×D; λi.<i,d>)  by Analyze THEN Reduce Concl
| |
| 5. a1 : ℕ
| 6. a2 : ℕ
| 7. <a1,d> = <a2,d>
| ⊢  a1 = a2  by Analyze-1 THEN OnAllHyps Reduce
|
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. 2. ¬¬(∃∈D)
. 3. Unbounded(ℕ×D)
. ⊢  ∃∈D  by Witness3: 1
. |
. 4. ∃∈(ℕ1 inj→ ℕ×D)
. ⊢  ∃∈D  by 2 Times (New:f Analyze4)
. |
. 4. f : ℕ1→ℕ×D
. 5. Inj(ℕ1; ℕ×D; f)  [not for witness]
. ⊢  ∃∈D  by Witness: f(0).2
END
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