```
bij_imp_exists_inv2  THM

⊢  ∀f:(A→B). Bij(A; B; f) ⇒ (∃g:(B→A). InvFuns(A;B;f;g))
| by Repeat
|    Def of
|    InvFuns(A;B;f;g) |
|    Bij(A; B; f) |
|    Inj(A; B; f) |
|    Surj(A; B; f)
|
1. A : Type
2. B : Type
3. f : A→B
4. ∀a1,a2:A. f(a1) = f(a2) ⇒ a1 = a2
5. ∀b:B. ∃a:A. f(a) = b
⊢  ∃g:(B→A). (∀x:A. g(f(x)) = x) & (∀y:B. f(g(y)) = y)
| by DescribeFun: g by ∀b:B. ∃!a:A. f(a) = b THEN Witness: g
|
|\
| .....assertion .....
| ⊢  ∀b:B. ∃!a:A. f(a) = b  by SimilarTo: Hyp:5
| |
| 5. b : B
| 6. a : A
| 7. f(a) = b
| ⊢  a is the A s.t. f(a) = b  by Analyze THEN Rename8: a'
| |
| 8. a' : A
| 9. f(a') = b
| ⊢  a' = a  by BackThru: Hyp:4
|
.\
. 6. ∀b:B. ∃!a:A. f(a) = b
. 7. g : B→A
. 8. ∀b:B. f(g(b)) = b
. ⊢  (∀x:A. g(f(x)) = x) & (∀y:B. f(g(y)) = y)  by Analyze
. |
. 9. x : A
. ⊢  g(f(x)) = x  by BackThru: Hyp:4
. |
. ⊢  f(g(f(x))) = f(x)  by BackThru: Hyp:8
END
```