```
bij_imp_exists_inv_version2  THM

⊢  ∀f:(A→B). Bij(A; B; f) ⇒ (∃g:(B→A). InvFuns(A;B;f;g))
| by SideProof 1
|
1. A : Type
2. B : Type
3. f : A→B
4. Inj(A; B; f)
5. Surj(A; B; f)
⊢  ∃g:(B→A). InvFuns(A;B;f;g)
| by DescribeFun: g by ∀b:B. ∃!a:A. f(a) = b
|
|\
| .....assertion .....
| ⊢  ∀b:B. ∃!a:A. f(a) = b  by SimilarTo: Surj(A; B; f)
| |
| 5. b : B
| 6. a : A
| 7. f(a) = b
| ⊢  a is the A s.t. f(a) = b  by Analyze THEN Rename8: a'
| |
| 8. a' : A
| 9. f(a') = b
| ⊢  a' = a  by BackThru: Hyp:4
|
.\
. 6. ∀b:B. ∃!a:A. f(a) = b
. 7. g : B→A
. 8. ∀b:B. f(g(b)) = b
. ⊢  ∃g:(B→A). InvFuns(A;B;f;g)  by Witness: g
. |
. ⊢  InvFuns(A;B;f;g)  by Analyze
. |
. 9. x : A
. ⊢  g(f(x)) = x  by BackThru: Hyp:4
. |
. ⊢  f(g(f(x))) = f(x)  by BackThru: Hyp:8
END
```

```
bij_imp_exists_inv_version2 Side Proof 1  THM

⊢  ∀A,B:Type, f:(A→B). Bij(A; B; f) ⇒ (∃g:(B→A). InvFuns(A;B;f;g))
| by UnivCD
|
1. A : Type
2. B : Type
3. f : A→B
4. Bij(A; B; f)
⊢  ∃g:(B→A). InvFuns(A;B;f;g)
| by Compute4 ⌈Bij(A; B; f)⌉ *⇒ Inj(A; B; f) & Surj(A; B; f)
|    THEN
|    Analyze 4
|
4. Inj(A; B; f)
5. Surj(A; B; f)
⊢  ∃g:(B→A). InvFuns(A;B;f;g)  by <Unproved PREMISE>
END
```