```
card_nat_vs_nat_tuple  THM

⊢  ∀k:ℕ+. (ℕk→ℕ) ~ ℕ  by NSubsetInd Concl
|
|\
| .....basecase .....
| 1. k : ℤ
| 2. 0<k
| 3. 1 = k
| ⊢  (ℕk→ℕ) ~ ℕ
|   by Rewrite by
|      Thm*  ∀a,a':ℤ.
|            a'-a = 1
|            ⇒
|            (∀B:({a..a'⁻}→Type{i}). (i:{a..a'⁻}→B(i)) ~ B(a))
|      {card_prod_family_intseg_singleton_elim}
|
.\
. .....upcase .....
. 1. k : ℤ
. 2. 1<k
. 3. (ℕ(k-1)→ℕ) ~ ℕ
. ⊢  (ℕk→ℕ) ~ ℕ  by Rewrite by (ℕk→ℕ) ~ (ℕ×ℕ)
. |
. |\
. | .....assertion .....
. | ⊢  (ℕk→ℕ) ~ (ℕ×ℕ)
. | | by Rewrite by
. | |    Thm*  ∀a,b:ℤ, c:{a...b}, B:({a..b⁻}→Type).
. | |          (i:{a..b⁻}→B(i))
. | |          ~
. | |          ((i:{a..c⁻}→B(i))×(i:{c..b⁻}→B(i)))
. | |    {card_split_prod_intseg_family}
. | |    Using:[a:= 0 | b:= k | c:= k-1]
. | |
. | ⊢  ((ℕ(k-1)→ℕ)×({(k-1)..k⁻}→ℕ)) ~ (ℕ×ℕ)  by Rewrite by Hyp:3
. | |
. | ⊢  (ℕ×({(k-1)..k⁻}→ℕ)) ~ (ℕ×ℕ)
. |   by Rewrite by
. |      Thm*  ∀a,a':ℤ.
. |            a'-a = 1
. |            ⇒
. |            (∀B:({a..a'⁻}→Type{i}). (i:{a..a'⁻}→B(i)) ~ B(a))
. |      {card_prod_family_intseg_singleton_elim}
. |
. .\
. . ⊢  (ℕ×ℕ) ~ ℕ
. .   by Thm*  (ℕ×ℕ) ~ ℕ    {card_nat_vs_nat_nat}  Asserted
END
```