```
card_nat_vs_nat_tuples_all  THM

⊢  (k:ℕ×ℕk→ℕ) ~ ℕ
| by Rewrite after
|    Inst: Thm: card_split_sigma_dom_decbl
|    Using:[A:= ℕ | B(k):= ℕk→ℕ | P(k):= 0 = k ∈ ℕ]
|
⊢  ((k:{k:ℕ| 0 = k ∈ ℕ }×ℕk→ℕ)+(k:{k:ℕ| ¬0 = k ∈ ℕ }×ℕk→ℕ)) ~ ℕ
| by ml_RWH_ooc_chain{:s, 0:s, true:s}
|                     ((k:{k:ℕ| 0 = k ∈ ℕ }×ℕk→ℕ |
|                      k:ℕ1×ℕk→ℕ |
|                      ℕ0→ℕ |
|                      ℕ1))
|    THEN
|    ml_RWH_ooc_chain{:s, 0:s, true:s}
|                     ((k:{k:ℕ| ¬0 = k ∈ ℕ }×ℕk→ℕ | k:ℕ+×ℕk→ℕ | ℕ))
|
|\
| .....assertion .....
| ⊢  (k:{k:ℕ| 0 = k ∈ ℕ }×ℕk→ℕ) ~ (k:ℕ1×ℕk→ℕ)
|   by OOCifExteq THEN Analyze
|
|\
| .....assertion .....
| ⊢  (k:ℕ1×ℕk→ℕ) ~ (ℕ0→ℕ)
|   by Rewrite by
|      Thm*  ∀a,a':ℤ.
|            a'-a = 1
|            ⇒
|            (∀B:({a..a'⁻}→Type{i}). (i:{a..a'⁻}×B(i)) ~ B(a))
|      {card_sum_family_intseg_singleton_elim}
|
|\
| .....assertion .....
| ⊢  (ℕ0→ℕ) ~ ℕ1
|   by BackThru: Thm*  (ℕ0→A) ~ ℕ1    {card_void_dom_fun_unit}
|
|\
| .....assertion .....
| ⊢  (k:{k:ℕ| ¬0 = k ∈ ℕ }×ℕk→ℕ) ~ (k:ℕ+×ℕk→ℕ)
| | by OOCifExteq THEN Analyze
| |
| 1. x : k:{k:ℕ| ¬0 = k ∈ ℕ }×ℕk→ℕ
| ⊢  x ∈ k:ℕ+×ℕk→ℕ  by 2 Times Analyze1
|
|\
| .....assertion .....
| ⊢  (k:ℕ+×ℕk→ℕ) ~ ℕ
|   by BackThru: Thm*  (k:ℕ+×ℕk→ℕ) ~ ℕ    {card_nat_vs_nat_tuples}
|
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. ⊢  (ℕ1+ℕ) ~ ℕ
.   by BackThru: Thm*  ∀k:ℕ. (ℕk+ℕ) ~ ℕ    {fin_plus_nat_ooc_nat}
END
```