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card_pi_vs_nsub_pi  THM

⊢  ∀a:ℕ, b:(ℕa→ℕ). (i:ℕa→ℕb(i)) ∼ ℕ(Π i:ℕa. b(i))  by NatInd Concl
|
|\
| 1. b : ℕ0→ℕ
| ⊢  (i:ℕ0→ℕb(i)) ∼ ℕ(Π i:ℕ0. b(i))  by OOCinterpolate ℕ1
| |
| |\
| | .....assertion .....
| | ⊢  (i:ℕ0→ℕb(i)) ∼ ℕ1
| | | by BackThru:
| | |    Thm*  (A ∼ ℕ1) ⟺ (∃!∈A)    {card1_iff_inhabited_uniquely}
| | |
| | ⊢  ∃!∈(i:ℕ0→ℕb(i))  by Witness: λx.x
| | |
| | 2. y : i:ℕ0→ℕb(i)
| | ⊢  (λx.x) = y  by FunExtensionality
| |
| .\
| . .....assertion .....
| . ⊢  ℕ1 ∼ ℕ(Π i:ℕ0. b(i))  by Compute ⌈Π i:ℕ0. b(i)⌉ *⇒ 1
|
.\
. 1. a : ℤ
. 2. 0<a
. 3. ∀b:(ℕ(a-1)→ℕ). (i:ℕ(a-1)→ℕb(i)) ∼ ℕ(Π i:ℕ(a-1). b(i))
. 4. b : ℕa→ℕ
. ⊢  (i:ℕa→ℕb(i)) ∼ ℕ(Π i:ℕa. b(i))
. | by Use:[a:= 0 | b:= a | c:= a-1]
. |    Rewrite by
. |    Thm*  ∀a,b:ℤ, c:{a...b}, B:({a..b⁻}→Type).
. |          (i:{a..b⁻}→B(i)) ∼ ((i:{a..c⁻}→B(i))×(i:{c..b⁻}→B(i)))
. |    {card_split_prod_intseg_family}
. |
. ⊢  ((i:ℕ(a-1)→ℕb(i))×(i:{(a-1)..a⁻}→ℕb(i))) ∼ ℕ(Π i:ℕa. b(i))
. | by Rewrite by
. |    Thm*  ∀f:(A→A→A), u:A, a,b:ℤ, e:({a..b⁻}→A).
. |          a<b
. |          ⇒
. |          (Iter(f;u) i:{a..b⁻}. e(i))
. |          =
. |          f(Iter(f;u) i:{a..b-1⁻}. e(i),e(b-1))
. |    {iter_via_intseg_split_last}
. |    THEN
. |    Reduce 0
. |
. ⊢  ((i:ℕ(a-1)→ℕb(i))×(i:{(a-1)..a⁻}→ℕb(i)))
. |  ∼
. |  ℕ((Π i:ℕ(a-1). b(i))•b(a-1))
. | by Rewrite by Hyp:3
. |    THEN
. |    Rewrite by
. |    Thm*  ∀a,a':ℤ.
. |          a'-a = 1
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. |          ⇒
. |          (∀B:({a..a'⁻}→Type{i}). (i:{a..a'⁻}→B(i)) ∼ B(a))
. |    {card_prod_family_intseg_singleton_elim}
. |
. ⊢  (ℕ(Π i:ℕ(a-1). b(i))×ℕb(a-1)) ∼ ℕ((Π i:ℕ(a-1). b(i))·b(a-1))
. | by BackThru: Thm*  ∀a,b:ℕ. (ℕa×ℕb) ∼ ℕ(a·b)    {nsub_mul_rw}
. |
. 5. x : ℕ
. 6. y : ℕ
. ⊢  x·y ∈ ℕ
.   by Auto THEN BackThru: Thm*  ∀a,b:ℕ. 0≤a·b    {mul_bounds_1a}
END
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