```
card_prod_family_singleton_elim  THM

⊢  (i:{a:A}→B(i)) ∼ B(a)  by Auto
|
1. A : Type
2. a : A
3. B : {a:A}→Type
⊢  (i:{a:A}→B(i)) ∼ B(a)  by a ∈ {a:A}  By Analyze
|
4. a ∈ {a:A}
⊢  (i:{a:A}→B(i)) ∼ B(a)
| by Witness: λf.f(a)
|    THEN
|    Witness: λb,i. b with type B(a)→(i:{a:A}→B(i))
|
|\
| .....assertion .....
| ⊢  (λb,i. b) ∈ B(a)→(i:{a:A}→B(i))  by 2 Times Analyze
| |
| 5. b : B(a)
| 6. i : {a:A}
| ⊢  b ∈ B(i)  by ChangeToEqType: B(a)
| |
| .....equality .....
| ⊢  B(i) = B(a) ∈ Type  by Analyze
| |
| ⊢  i = a
|   by BackThru:
|      Thm*  ∀x:{a:A}. x = a ∈ {a:A}    {singleton_eq_in_self}
|
.\
. 5. (λb,i. b) ∈ B(a)→(i:{a:A}→B(i))
. ⊢  InvFuns(i:{a:A}→B(i);B(a);λf.f(a);λb,i. b)
. | by Analyze THEN Reduce Concl THEN Rename-1: f
. |
. 6. f : i:{a:A}→B(i)
. ⊢  (λi.f(a)) = f  by FunExtensionality
. |
. |\
. | 7. x : {a:A}
. | ⊢  f(a) = f(x) ∈ B(x)  by ApFun: f to: x = a ∈ {a:A}
. | |
. | .....assertion .....
. | ⊢  x = a
. |   by BackThru:
. |      Thm*  ∀x:{a:A}. x = a ∈ {a:A}    {singleton_eq_in_self}
. |
. .\
. . .....wf .....
. . ⊢  (λi.f(a)) ∈ i:{a:A}→B(i)  by Analyze
. . |
. . 7. i : {a:A}
. . ⊢  f(a) ∈ B(i)  by ApFun: f to: i = a ∈ {a:A}
. . |
. . .....assertion .....
```

```
. . ⊢  i = a
. .   by BackThru:
. .      Thm*  ∀x:{a:A}. x = a ∈ {a:A}    {singleton_eq_in_self}
END
```