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card_sigma  THM

⊢  (x:A. B(x)) ∼ (x:A'. B'(x)) ⇒ ((x:A×B(x)) ∼ (x:A'×B'(x)))
| by Def of (x:A. B(x)) ∼ (x:A'. B'(x)) THEN ExistHD Hyp:-1
|
1. A : Type
2. A' : Type
3. B : A→Type
4. B' : A'→Type
5. f : A→A'
6. g : A'→A
7. F : x:A→B(x)→B'(f(x))
8. G : x:A→B'(f(x))→B(x)
9. InvFuns(A;A';f;g)
10. ∀u:A. InvFuns(B(u);B'(f(u));F(u);G(u))
⊢  (x:A×B(x)) ∼ (x:A'×B'(x))
| by Witness: λe.e/x,y. <f(x),F(x,y)> | λe.e/x,y. <g(x),G(g(x),y)>
|    THENA
|    (ApFun: B' to: f(g(x)) = x ∈ A' THENA BackThru: Hyp:9)
|
⊢  InvFuns(x:A×B(x);x:A'×B'(x)
|          ;λe.e/x,y. <f(x),F(x,y)>;λe.e/x,y. <g(x),G(g(x),y)>)
| by Analyze
|    THEN
|    Rename-1: e THEN New:[x | y] Analyze-1 THEN Reduce Concl
|
|\
| 11. x : A
| 12. y : B(x)
| ⊢  <g(f(x)),G(g(f(x)),F(x,y))> = <x,y> ∈ x:A×B(x)
| | by g(f(x)) = x ∈ A  By BackThru: Hyp:9
| |    THEN
| |    ApFun: λx.B'(f(x)) to: g(f(x)) = x ∈ A
| |    THEN
| |    Rewrite by g(f(x)) = x ∈ A
| |
| 13. g(f(x)) = x
| 14. B'(f(g(f(x)))) = B'(f(x))
| ⊢  <x,G(x,F(x,y))> = <x,y> ∈ x:A×B(x)  by Analyze
| |
| ⊢  G(x,F(x,y)) = y  by BackThru: Hyp:10
|
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. 11. x : A'
. 12. y : B'(x)
. ⊢  <f(g(x)),F(g(x),G(g(x),y))> = <x,y> ∈ x:A'×B'(x)
. | by f(g(x)) = x ∈ A'  By BackThru: Hyp:9
. |    THEN
. |    ApFun: B' to: f(g(x)) = x ∈ A'
. |    THEN
. |    Rewrite by f(g(x)) = x ∈ A'
. |
. 13. f(g(x)) = x
. 14. B'(f(g(x))) = B'(x)
. ⊢  <x,F(g(x),G(g(x),y))> = <x,y> ∈ x:A'×B'(x)  by Analyze
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. ⊢  F(g(x),G(g(x),y)) = y ∈ B'(x)  by BackThru: Hyp:10
END
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