```
card_sigma_vs_nsub_sigma  THM

⊢  ∀a:ℕ, b:(ℕa→ℕ). (i:ℕa×ℕb(i)) ~ ℕ(Σ i:ℕa. b(i))  by NatInd Concl
|
|\
| 1. b : ℕ0→ℕ
| ⊢  (i:ℕ0×ℕb(i)) ~ ℕ(Σ i:ℕ0. b(i))  by OOCinterpolate ℕ0
| |
| |\
| | .....assertion .....
| | ⊢  (i:ℕ0×ℕb(i)) ~ ℕ0
| | | by BackThru: Thm*  (A ~ ℕ0) ⟺ ¬(∃∈A)    {card0_iff_uninhabited}
| | |    THEN
| | |    Analyze
| | |
| | 2. ∃∈(i:ℕ0×ℕb(i))
| | ⊢  False  by 2 Times Analyze-1
| |
| .\
| . .....assertion .....
| . ⊢  ℕ0 ~ ℕ(Σ i:ℕ0. b(i))  by Compute ⌜Σ i:ℕ0. b(i)⌝ *⇒ 0
|
.\
. 1. a : ℤ
. 2. 0<a
. 3. ∀b:(ℕ(a-1)→ℕ). (i:ℕ(a-1)×ℕb(i)) ~ ℕ(Σ i:ℕ(a-1). b(i))
. 4. b : ℕa→ℕ
. ⊢  (i:ℕa×ℕb(i)) ~ ℕ(Σ i:ℕa. b(i))
. | by Use:[a:= 0 | b:= a | c:= a-1]
. |    Rewrite by
. |    Thm*  ∀a,b:ℤ, c:{a...b}, B:({a..b⁻}→Type).
. |          (i:{a..b⁻}×B(i)) ~ ((i:{a..c⁻}×B(i))+(i:{c..b⁻}×B(i)))
. |    {card_split_sum_intseg_family}
. |
. ⊢  ((i:ℕ(a-1)×ℕb(i))+(i:{(a-1)..a⁻}×ℕb(i))) ~ ℕ(Σ i:ℕa. b(i))
. | by Rewrite by
. |    Thm*  ∀f:(A→A→A), u:A, a,b:ℤ, e:({a..b⁻}→A).
. |          a<b
. |          ⇒
. |          (Iter(f;u) i:{a..b⁻}. e(i))
. |          =
. |          f(Iter(f;u) i:{a..b-1⁻}. e(i),e(b-1))
. |    {iter_via_intseg_split_last}
. |    THEN
. |    Reduce 0
. |
. ⊢  ((i:ℕ(a-1)×ℕb(i))+(i:{(a-1)..a⁻}×ℕb(i)))
. |  ~
. |  ℕ((Σ i:ℕ(a-1). b(i))+b(a-1))
. | by Rewrite by Hyp:3
. |    THEN
. |    Rewrite by
. |    Thm*  ∀a,a':ℤ.
. |          a'-a = 1
. |          ⇒
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. |           (∀B:({a..a'⁻}→Type{i}). (i:{a..a'⁻}×B(i)) ~ B(a))
. |     {card_sum_family_intseg_singleton_elim}
. |
. ⊢  (ℕ(Σ i:ℕ(a-1). b(i))+ℕb(a-1)) ~ ℕ((Σ i:ℕ(a-1). b(i))+b(a-1))
.   by BackThru: Thm*  ∀a,b:ℕ. (ℕa+ℕb) ~ ℕ(a+b)    {nsub_add_rw}
END
```