```
card_split_decbl  THM

⊢  (∀x:A. Dec(P(x))) ⇒ (A ~ ({x:A| P(x) }+{x:A| ¬P(x) }))  by UnivCD
|
1. A : Type
2. P : A→Prop
3. ∀x:A. Dec(P(x))
⊢  A ~ ({x:A| P(x) }+{x:A| ¬P(x) })
| by Rewrite'-1 by
|    Thm*  Dec(P) ⟺ (∃!i:ℕ2. if i=_2 0→ P else ¬P fi)
|    {decidable_vs_unique_nsub2}
|
3. ∀x:A. ∃!i:ℕ2. if i=_2 0→ P(x) else ¬P(x) fi
⊢  A ~ ({x:A| P(x) }+{x:A| ¬P(x) })
| by Rewrite by
|    Thm*  (A+B) ~ (i:ℕ2×if i=_2 0→ A else B fi)
|    {card_union_vs_sigma}
|
⊢  A ~ (i:ℕ2×if i=_2 0→ {x:A| P(x) } else {x:A| ¬P(x) } fi)
| by FwdThru:
|    Thm*  ∀P:(A→B→Prop).
|           (∀x:A. ∃!y:B. P(x;y)) ⇒ (A ~ (y:B×{x:A| P(x;y) }))
|    {partition_type}
|    on [ Hyp:-1 ]
|
4. A ~ (i:ℕ2×{x:A| if i=_2 0→ P(x) else ¬P(x) fi })
⊢  A ~ (i:ℕ2×if i=_2 0→ {x:A| P(x) } else {x:A| ¬P(x) } fi)
| by Rewrite by Hyp:-1
|
⊢  (i:ℕ2×{x:A| if i=_2 0→ P(x) else ¬P(x) fi })
|  ~
|  (i:ℕ2×if i=_2 0→ {x:A| P(x) } else {x:A| ¬P(x) } fi)
| by BackThru:
|    Thm*  ∀B,B':(A→Type).
|           (∀x:A. B(x) ~ B'(x)) ⇒ ((x:A×B(x)) ~ (x:A×B'(x)))
|    {product_functionality_wrt_one_one_corr_B}
|    THEN
|    CBV: {x:A| <*> }
|
5. i : ℕ2
⊢  {x:A| if i=_2 0→ P(x) else ¬P(x) fi }
   ~
   if i=_2 0→ {x:A| P(x) } else {x:A| ¬P(x) } fi
  by Rewrite← by
     Thm*  {x:A| if b→ P(x) else Q(x) fi }
           ~
           if b→ {x:A| P(x) } else {x:A| Q(x) } fi
     {ifthenelse_distr_subtype_ooc}
END
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