```
card_split_prod_intseg_family  THM

⊢  ∀a,b:ℤ, c:{a...b}, B:({a..b⁻}→Type).
|  (i:{a..b⁻}→B(i)) ∼ ((i:{a..c⁻}→B(i))×(i:{c..b⁻}→B(i)))  by Auto
|
1. a : ℤ
2. b : ℤ
3. c : {a...b}
4. B : {a..b⁻}→Type
⊢  (i:{a..b⁻}→B(i)) ∼ ((i:{a..c⁻}→B(i))×(i:{c..b⁻}→B(i)))
| by Use:[A:= {a..b⁻} | P(i):= i<c]
|    Inst:
|    Thm*  (∀x:A. Dec(P(x)))
|          ⇒
|          ((x:A→B(x)) ∼ ((x:A st P(x)→B(x))×(x:A st ¬P(x)→B(x))))
|    {card_split_prod_intseg_family_decbl}
|    THEN
|    Rewrite by Hyp:-1
|
⊢  ((i:{a..b⁻} st i<c→B(i))×(i:{a..b⁻} st ¬i<c→B(i)))
|  ∼
|  ((i:{a..c⁻}→B(i))×(i:{c..b⁻}→B(i)))
| by BackThru:
|    Thm*  (A ∼ A') ⇒ (B ∼ B') ⇒ ((A×B) ∼ (A'×B'))
|    {product_functionality_wrt_one_one_corr}
|    THEN
|    OOCifExteq
|
|\
| ⊢  (i:{a..b⁻} st i<c→B(i)) =ext (i:{a..c⁻}→B(i))  by Analyze
|
.\
. ⊢  (i:{a..b⁻} st ¬i<c→B(i)) =ext (i:{c..b⁻}→B(i))  by Analyze
END
```