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card_split_sum_intseg_family  THM

⊢  ∀a,b:ℤ, c:{a...b}, B:({a..b⁻}→Type).
|  (i:{a..b⁻}×B(i)) ~ ((i:{a..c⁻}×B(i))+(i:{c..b⁻}×B(i)))  by Auto
|
1. a : ℤ
2. b : ℤ
3. c : {a...b}
4. B : {a..b⁻}→Type
⊢  (i:{a..b⁻}×B(i)) ~ ((i:{a..c⁻}×B(i))+(i:{c..b⁻}×B(i)))
| by Inst:
|    Thm*  ∀B:(A→Type), P:(A→Prop).
|          (∀i:A. Dec(P(i)))
|          ⇒
|          ((i:A×B(i))
|           ~
|           ((i:{i:A| P(i) }×B(i))+(i:{i:A| ¬P(i) }×B(i))))
|    {card_split_sigma_dom_decbl}
|    Using:[A:= {a..b⁻} | B:= B | P(i):= i<c]
|    THEN
|    Rewrite by Hyp:-1
|
5. (i:{a..b⁻}×B(i))
|  ~
|  ((i:{i:{a..b⁻}| i<c }×B(i))+(i:{i:{a..b⁻}| ¬i<c }×B(i)))
⊢  ((i:{i:{a..b⁻}| i<c }×B(i))+(i:{i:{a..b⁻}| ¬i<c }×B(i)))
|  ~
|  ((i:{a..c⁻}×B(i))+(i:{c..b⁻}×B(i)))
| by BackThru:
|    Thm*  (A ~ A') ⇒ (B ~ B') ⇒ ((A+B) ~ (A'+B'))
|    {union_functionality_wrt_one_one_corr}
|
|\
| ⊢  (i:{i:{a..b⁻}| i<c }×B(i)) ~ (i:{a..c⁻}×B(i))
|   by OOCifExteq THEN Analyze
|
.\
. ⊢  (i:{i:{a..b⁻}| ¬i<c }×B(i)) ~ (i:{c..b⁻}×B(i))
.   by OOCifExteq THEN Analyze
END
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