```
card_st_vs_msize  THM

⊢  ∀f:(ℕa→ℕ2).
|  (∀i:ℕa. P(i) ⟺ f(i) = 1 ∈ ℕ2) ⇒ ({x:ℕa| P(x) } ~ ℕ(Msize(f)))
| by Def of Msize(ℕ<nat>) THEN Reduce Concl
|     THEN
|     Rewrite by
|     Thm*  ∀a:ℕ, b:(ℕa→ℕ). (i:ℕa×ℕb(i)) ~ ℕ(Σ i:ℕa. b(i))
|     {card_sigma_vs_nsub_sigma}
|
⊢  ∀a:ℕ, P:(ℕa→Prop), f:(ℕa→ℕ2).
|  (∀i:ℕa. P(i) ⟺ f(i) = 1 ∈ ℕ2) ⇒ ({x:ℕa| P(x) } ~ (i:ℕa×ℕ(f(i))))
| by Auto
|
1. a : ℕ
2. P : ℕa→Prop
3. f : ℕa→ℕ2
4. ∀i:ℕa. P(i) ⟺ f(i) = 1 ∈ ℕ2
⊢  {x:ℕa| P(x) } ~ (i:ℕa×ℕ(f(i)))
| by Witness:
|       (λx.<x,0>) ∈ {x:ℕa| P(x) }→(i:ℕa×ℕ(f(i))) |
|       (λxi.xi.1) ∈ (i:ℕa×ℕ(f(i)))→{x:ℕa| P(x) }
|
|\
| 5. x : {x:ℕa| P(x) }
| ⊢  0 ∈ ℕ(f(x))  by Analyze-1
| |
| 5. x : ℕa
| 6. P(x)
| ⊢  0 ∈ ℕ(f(x))  by FwdThru: Hyp:4 on [ P(x) ]
|
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| 5. (λx.<x,0>) ∈ {x:ℕa| P(x) }→(i:ℕa×ℕ(f(i)))
| 6. xi : i:ℕa×ℕ(f(i))
| 7. x : ℕa
| 8. i : ℕ(f(x))
| 9. xi = <x,i> ∈ i:ℕa×ℕ(f(i))
| ⊢  x ∈ {x:ℕa| P(x) }  by Analyze
| |
| .....set predicate .....
| ⊢  P(x)  by BackThru: Hyp:4
|
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. 5. (λx.<x,0>) ∈ {x:ℕa| P(x) }→(i:ℕa×ℕ(f(i)))
. 6. (λxi.xi.1) ∈ (i:ℕa×ℕ(f(i)))→{x:ℕa| P(x) }
. ⊢  InvFuns({x:ℕa| P(x) };i:ℕa×ℕ(f(i));λx.<x,0>;λxi.xi.1)
. | by Analyze THEN Reduce Concl
. |
. 7. y : i:ℕa×ℕ(f(i))
. ⊢  <y.1,0> = y ∈ i:ℕa×ℕ(f(i))  by Analyze-1 THEN Reduce Concl
. |
. 7. i : ℕa
. 8. y1 : ℕ(f(i))
. ⊢  <i,0> = <i,y1> ∈ i:ℕa×ℕ(f(i))  by Analyze
END
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