```
compose_iter_inj_cycles  THM

⊢  ∀k:ℕ, f:(ℕk inj→ ℕk), u:ℕk. ∃i:ℕ+. f{^0i}(u) = u  by Autoℕ
|
1. k : ℕ
2. f : ℕk inj→ ℕk
3. u : ℕk
⊢  ∃i:ℕ+. f{^0i}(u) = u  by Analyze-2
|
2. f : ℕk→ℕk
3. Inj(ℕk; ℕk; f)
4. u : ℕk
⊢  ∃i:ℕ+. f{^0i}(u) = u
| by ∃i,j:ℕ. i<j & f{^0j}(u) = f{^0i}(u) ∈ ℕk  Asserted
|
|\
| .....assertion .....
| ⊢  ∃i,j:ℕ. i<j & f{^0j}(u) = f{^0i}(u)
| | by Inst:
| |    Thm*  ∀m,k:ℕ, f:(ℕm→ℕk). k<m ⇒ (∃x,y:ℕm. x ≠ y & f(x) = f(y))
| |    {pigeon_hole}
| |    Using:[k+1 | k | λi.f{^0i}(u)] ...aℕ
| |    THEN
| |    Reduce Hyp:-1
| |
| 5. ∃x,y:ℕ(k+1). x ≠ y & f{^0x}(u) = f{^0y}(u)
| ⊢  ∃i,j:ℕ. i<j & f{^0j}(u) = f{^0i}(u)
| | by SimilarTo: ∃i,j:ℕ(k+1). i<j & f{^0i}(u) = f{^0j}(u) ∈ ℤ ...ℕ
| |
| .....assertion .....
| ⊢  ∃i,j:ℕ(k+1). i<j & f{^0i}(u) = f{^0j}(u)  by ExistHD Hyp:5
| |
| 5. x : ℕ(k+1)
| 6. y : ℕ(k+1)
| 7. x ≠ y
| 8. f{^0x}(u) = f{^0y}(u)
| ⊢  ∃i,j:ℕ(k+1). i<j & f{^0i}(u) = f{^0j}(u)  by Decide: x<y ...wℕ
| |
| |\
| | 9. x<y
| | ⊢  ∃i,j:ℕ(k+1). i<j & f{^0i}(u) = f{^0j}(u)  by Witness: x | y ...ℕ
| |
| .\
| . 9. ¬x<y
| . ⊢  ∃i,j:ℕ(k+1). i<j & f{^0i}(u) = f{^0j}(u)  by Witness: y | x ...ℕ
|
.\
. 5. ∃i,j:ℕ. i<j & f{^0j}(u) = f{^0i}(u)
. ⊢  ∃i:ℕ+. f{^0i}(u) = u  by ExistHD Hyp:-1
. |
. 5. i : ℕ
. 6. j : ℕ
. 7. i<j
. 8. f{^0j}(u) = f{^0i}(u)
. ⊢  ∃i:ℕ+. f{^0i}(u) = u
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. | by Rewrite-1 by
. |    Thm*  ∀x:A, f:(A→A), k:ℕ, i:{0...k}.
. |          f{⁰k}(x) = f{⁰i}(f{⁰k-i}(x))
. |    {compose_iter_sum_rw}
. |    Using:[A:= ℕk | k:= j | i:= i | x:= u] ...ℕ
. |
. 9. f{⁰i}(f{⁰j-i}(u)) = f{⁰i}(u)
. ⊢  ∃i:ℕ+. f{⁰i}(u) = u
. | by FwdThru:
. |    Thm*  Inj(A; A; f) ⇒ (∀i:ℕ. Inj(A; A; f{⁰i}))
. |    {compose_iter_injection}
. |    on [ Hyp:-1 ] ...ℕ
. |
. 10. f{⁰j-i}(u) = u
. ⊢  ∃i:ℕ+. f{⁰i}(u) = u  by Witness: j-i ...ℕ
END
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