```
compose_iter_injection  THM

⊢  Inj(A; A; f) ⇒ (∀i:ℕ. Inj(A; A; f{^0i}))  by Auto
|
1. A : Type
2. f : A→A
3. Inj(A; A; f)
4. i : ℕ
⊢  Inj(A; A; f{^0i})  by NatInd Hyp:-1
|
|\
| .....basecase .....
| 3. Inj(A; A; f)
| ⊢  Inj(A; A; f{^00})
| | by Rewrite by
| |    Thm*  ∀f:(A→A). f{^00} = Id    {compose_iter_zero_id}
| |
| ⊢  Inj(A; A; Id)  by Def THEN Reduce Concl
|
.\
. .....upcase .....
. 4. i : ℤ
. 5. 0<i
. 6. Inj(A; A; f{^0i-1})
. ⊢  Inj(A; A; f{^0i})  by SimilarTo: Hyp:-1
. |
. 6. a1 : A
. 7. a2 : A
. 8. f{^0i}(a1) = f{^0i}(a2)
. ⊢  f{^0i-1}(a1) = f{^0i-1}(a2)
. | by Compute-1
. |      (⌈⌈f{^0i}⌉(a1)⌉ *⇒ if i=_2 0→ a1 else f(f{^0i-1}(a1)) fi)
. |      =
. |      (⌈⌈f{^0i}⌉(a2)⌉ *⇒ if i=_2 0→ a2 else f(f{^0i-1}(a2)) fi)
. |      ∈ A
. |    THEN
. |    SplitITE Hyp:-1
. |
. .....falsecase .....
. 8. f(f{^0i-1}(a1)) = f(f{^0i-1}(a2))
. 9. i ≠ 0
. ⊢  f{^0i-1}(a1) = f{^0i-1}(a2)  by BackThru: Inj(A; A; f)
END
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