```
compose_iter_inverses  THM

⊢  InvFuns(A;A;f;g) ⇒ (∀i:ℕ. InvFuns(A;A;f{^Oi};g{^Oi}))  by Analyze
|
1. A : Type
⊢  ∀f,g:(A→A). InvFuns(A;A;f;g) ⇒ (∀i:ℕ. InvFuns(A;A;f{^Oi};g{^Oi}))
| by ∀f,g:(A→A). InvFuns(A;A;f;g) ⇒ (∀i:ℕ, x:A. g{^Oi}(f{^Oi}(x)) = x)
|    Asserted
|
|\
| 2. f : A→A
| 3. g : A→A
| 4. InvFuns(A;A;f;g)
| 5. i : ℕ
| 6. x : A
| ⊢  g{^Oi}(f{^Oi}(x)) = x  by NatInd Hyp:-2
| |
| |\
| | .....basecase .....
| | 5. x : A
| | ⊢  g{^O0}(f{^O0}(x)) = x  by Compute ⌈g{^O0}(f{^O0}(x))⌉ *⇒ x
| |
| .\
| . .....upcase .....
| . 5. x : A
| . 6. i : ℤ
| . 7. 0<i
| . 8. g{^Oi-1}(f{^Oi-1}(x)) = x
| . ⊢  g{^Oi}(f{^Oi}(x)) = x
| . | by Rewrite by g{^Oi}(f{^Oi}(x)) = g{^Oi-1}(g(f(f{^Oi-1}(x)))) ∈ A
| . |
| . |\
| . | .....assertion .....
| . | ⊢  g{^Oi}(f{^Oi}(x)) = g{^Oi-1}(g(f(f{^Oi-1}(x))))
| . | | by Thm*  ∀x:A, f:(A→A), i,j,k:ℕ.
| . | |          k = i+j ⇒ f{^Oi}(f{^Oj}(x)) = f{^Ok}(x)
| . | |    {compose_iter_sum}
| . | |
| . | ⊢  g{^Oi-1}(g{^O1}(f{^O1}(f{^Oi-1}(x)))) = g{^Oi-1}(g(f(f{^Oi-1}(x))))
| . |   by Rewrite by
| . |      Thm*  ∀f:(A→A). f{^O1} = f    {compose_iter_once}
| . |
| . .\
| . . ⊢  g{^Oi-1}(g(f(f{^Oi-1}(x)))) = x
| . .   by Analyze4 THEN Rewrite by ∀x:A. g(f(x)) = x
|
.\
. 2. ∀f,g:(A→A). InvFuns(A;A;f;g) ⇒ (∀i:ℕ, x:A. g{^Oi}(f{^Oi}(x)) = x)
. 3. f : A→A
. 4. g : A→A
. 5. InvFuns(A;A;f;g)
. 6. i : ℕ
. ⊢  InvFuns(A;A;f{^Oi};g{^Oi})  by Analyze
. |
. |\
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. | 7. x : A
. | ⊢  g{^Oi}(f{^Oi}(x)) = x  by BackThru: Hyp:2
. |
. .\
. . 7. y : A
. . ⊢  f{^Oi}(g{^Oi}(y)) = y  by BackThru: Hyp:2
. . |
. . ⊢  InvFuns(A;A;g;f)
. .   by BackThru:
. .      Thm*  InvFuns(A;B;f;g) ⇒ InvFuns(B;A;g;f)
. .      {inv_funs_2_sym}
END
```