```
compose_iter_surjection  THM

⊢  Surj(A; A; f) ⇒ (∀i:ℕ. Surj(A; A; f{^o i}))  by Auto
|
1. A : Type
2. f : A→A
3. Surj(A; A; f)
4. i : ℕ
⊢  Surj(A; A; f{^o i})  by NatInd Hyp:-1
|
|\
| .....basecase .....
| 3. Surj(A; A; f)
| ⊢  Surj(A; A; f{^o 0})
| | by Rewrite by
| |    Thm*  ∀f:(A→A). f{^o 0} = Id    {compose_iter_zero_id}
| |
| ⊢  Surj(A; A; Id)  by Def THEN Reduce Concl
| |
| 4. b : A
| ⊢  ∃a:A. a = b  by Witness: b
|
.\
. .....upcase .....
. 4. i : ℤ
. 5. 0<i
. 6. Surj(A; A; f{^o i-1})
. ⊢  Surj(A; A; f{^o i})  by SimilarTo Hyp:-1
. |
. 6. b : A
. 7. ∃a:A. f{^o i-1}(a) = b
. ⊢  ∃a:A. f{^o i}(a) = b  by Analyze-1
. |
. 7. a : A
. 8. f{^o i-1}(a) = b
. ⊢  ∃a:A. f{^o i}(a) = b
. | by Inst: Surj(A; A; f) Using:[a] THEN New:u Analyze-1
. |
. 9. u : A
. 10. f(u) = a
. ⊢  ∃a:A. f{^o i}(a) = b  by Witness: u THEN Rewrite8 by f(u) = a ∈ A
. |
. 11. f{^o i-1}(f(u)) = b
. ⊢  f{^o i}(u) = b  by Rewrite by Hyp:-1
. |
. ⊢  f{^o i}(u) = f{^o i-1}(f(u))
. | by Rewrite by f(u) = f{^o 1}(u) ∈ A
. |    THENA
. |    Rewrite by Thm*  ∀f:(A→A). f{^o 1} = f    {compose_iter_once}
. |
. ⊢  f{^o i}(u) = f{^o i-1}(f{^o 1}(u))
.   by BackThru:
.      Thm*  ∀x:A, f:(A→A), i,j,k:ℕ.
.            k = i+j ⇒ f{^o i}(f{^o j}(x)) = f{^o k}(x)
.      {compose_iter_sum}
```

```
END
```