```
counting_is_unique  THM

⊢  (A ∼ ℕm) ⇒ (A ∼ ℕk) ⇒ m = k  by Auto
|
1. A : Type
2. m : ℕ
3. k : ℕ
4. A ∼ ℕm
5. A ∼ ℕk
⊢  m = k  by ∀x,y:ℕ. (ℕx ∼ ℕy) ⇒ x≤y  Asserted
|
|\
| 6. x : ℕ
| 7. y : ℕ
| 8. ℕx ∼ ℕy
| ⊢  x≤y
| | by FwdThru:
| |    Thm*  (∃f:(A→B). Bij(A; B; f)) ⟺ (A ∼ B)
| |    {bij_iff_1_1_corr_version2} onHyps
| |    [Hyp:8]
| |    THEN
| |    BackThru:
| |    Thm*  (∃f:(ℕm→ℕk). Inj(ℕm; ℕk; f)) ⇒ m≤k    {inj_imp_le}
| |
| 9. ∃f:(ℕx→ℕy). Bij(ℕx; ℕy; f)
| ⊢  ∃f:(ℕx→ℕy). Inj(ℕx; ℕy; f)  by SimilarTo: Hyp:9
| |
| 9. f : ℕx→ℕy
| 10. Bij(ℕx; ℕy; f)
| ⊢  Inj(ℕx; ℕy; f)  by Analyze10
|
.\
. 6. ∀x,y:ℕ. (ℕx ∼ ℕy) ⇒ x≤y
. ⊢  m = k
.   by m≤k & k≤m  Asserted THEN BackThru: Hyp:6
.      THEN
.      ChainRelation:
.      Thm*  EquivRel X,Y:Type. X ∼ Y    {one_one_corr_is_equiv_rel}
END
```