```
delete_fenum_value_is_fenum  THM

⊢  Bij(ℕ(m+1); ℕ(k+1); f) ⇒ Bij(ℕm; ℕk; Replace value k by f(m) in f)
| by UnivCD THEN Analyze4
|
1. m : ℕ
2. k : ℕ
3. f : ℕ(m+1)→ℕ(k+1)
4. Inj(ℕ(m+1); ℕ(k+1); f)
5. Surj(ℕ(m+1); ℕ(k+1); f)
⊢  Bij(ℕm; ℕk; Replace value k by f(m) in f)  by Analyze
|
|\
| ⊢  Inj(ℕm; ℕk; Replace value k by f(m) in f)
|   by BackThru:
|      Thm*  Inj(ℕ(m+1); ℕ(k+1); f)
|            ⇒
|            Inj(ℕm; ℕk; Replace value k by f(m) in f)
|      {delete_fenum_value_is_inj}
|
.\
. ⊢  Surj(ℕm; ℕk; Replace value k by f(m) in f)  by Analyze
. |
. 6. b : ℕk
. ⊢  ∃a:ℕm. (Replace value k by f(m) in f)(a) = b
. | by Witness5: b THEN Analyze-1
. |    THEN
. |    Witness5: k THEN New:c Analyze-1
. |
. 7. a : ℕ(m+1)
. 8. f(a) = b ∈ ℕ(k+1)
. 9. c : ℕ(m+1)
. 10. f(c) = k ∈ ℕ(k+1)
. ⊢  ∃a:ℕm. (Replace value k by f(m) in f)(a) = b
. | by Decide: a = m ∈ ℕ(m+1)
. |
. |\
. | 11. a = m ∈ ℕ(m+1)
. | ⊢  ∃a:ℕm. (Replace value k by f(m) in f)(a) = b
. | | by Witness: c
. | |    THEN
. | |    Rewrite by
. | |    Thm*  Inj(ℕ(m+1); ℕ(k+1); f)
. | |          ⇒
. | |          (∀i:ℕm.
. | |           f(i) = k
. | |           ⇒
. | |           (Replace value k by f(m) in f)(i) = f(m) ∈ℕk)
. | |    {delete_fenum_value_comp1}
. | |
. | ⊢  f(m) = b ∈ ℕk  by Rewrite8 by Hyp:11
. |
. .\
. . 11. ¬a = m ∈ ℕ(m+1)
. . ⊢  ∃a:ℕm. (Replace value k by f(m) in f)(a) = b
```

```
. .   by Witness: a
. .      THEN
. .      Rewrite by
. .      Thm*  Inj(ℕ(m+1); ℕ(k+1); f)
. .            ⇒
. .            (∀i:ℕm.
. .             ¬f(i) = k
. .             ⇒
. .             (Replace value k by f(m) in f)(i) = f(i) ∈ℕk)
. .      {delete_fenum_value_comp2}
END
```