```
delete_fenum_value_is_fenum_gen  THM

⊢  Bij({u:ℤ| P(u) }; {v:ℤ| Q(v) }; f)
|  ⇒
|  (∀m:{u:ℤ| P(u) }, k:{v:ℤ| Q(v) }.
|   Bij({u:ℤ| P(u) & ¬u = m }; {v:ℤ| Q(v) & ¬v = k };
|       Replace value k by f(m) in f))  by UnivCD THEN Analyze4
|
1. P : ℤ→Prop
2. Q : ℤ→Prop
3. f : {u:ℤ| P(u) }→{v:ℤ| Q(v) }
4. Inj({u:ℤ| P(u) }; {v:ℤ| Q(v) }; f)
5. Surj({u:ℤ| P(u) }; {v:ℤ| Q(v) }; f)
6. m : {u:ℤ| P(u) }
7. k : {v:ℤ| Q(v) }
⊢  Bij({u:ℤ| P(u) & ¬u = m }; {v:ℤ| Q(v) & ¬v = k };
|      Replace value k by f(m) in f)  by Analyze
|
|\
| ⊢  Inj({u:ℤ| P(u) & ¬u = m }; {v:ℤ| Q(v) & ¬v = k };
|        Replace value k by f(m) in f)
|   by BackThru:
|      Thm*  Inj({u:ℤ| P(u) }; {v:ℤ| Q(v) }; f)
|            ⇒
|            (∀m:{u:ℤ| P(u) }, k:{v:ℤ| Q(v) }.
|             Inj({u:ℤ| P(u) & ¬u = m }; {v:ℤ| Q(v) & ¬v = k };
|                 Replace value k by f(m) in f))
|      {delete_fenum_value_is_inj_gen}
|
.\
. ⊢  Surj({u:ℤ| P(u) & ¬u = m }; {v:ℤ| Q(v) & ¬v = k };
. |       Replace value k by f(m) in f)
. | by (Replace value k by f(m) in f)
. |    ∈ {u:ℤ| P(u) & ¬u = m ∈ ℤ }→{v:ℤ| Q(v) & ¬v = k ∈ ℤ }
. |    Asserted
. |    THEN
. |    Analyze
. |
. 8. (Replace value k by f(m) in f)
. |  ∈ {u:ℤ| P(u) & ¬u = m }→{v:ℤ| Q(v) & ¬v = k }
. 9. b : {v:ℤ| Q(v) & ¬v = k }
. ⊢  ∃a:{u:ℤ| P(u) & ¬u = m }. (Replace value k by f(m) in f)(a) = b
. | by Witness5: b THEN Analyze-1
. |    THEN
. |    Witness5: k THEN New:c Analyze-1
. |
. 10. a : {u:ℤ| P(u) }
. 11. f(a) = b ∈ {v:ℤ| Q(v) }
. 12. c : {u:ℤ| P(u) }
. 13. f(c) = k
. ⊢  ∃a:{u:ℤ| P(u) & ¬u = m }. (Replace value k by f(m) in f)(a) = b
. | by Decide: a = m ∈ ℤ
. |
. |\
. | 14. a = m
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. | ⊢  ∃a:{u:ℤ| P(u) & ¬u = m }. (Replace value k by f(m) in f)(a) = b
. | | by Analyze12 THEN Witness: c
. | |
. | 12. c : ℤ
. | 13. P(c)
. | 14. f(c) = k
. | 15. a = m
. | ⊢  (Replace value k by f(m) in f)(c) = b
. | | by Rewrite by Thm: delete_fenum_value_comp1_gen
. | |
. | ⊢  f(m) = b ∈ {v:ℤ| Q(v) & ¬v = k }
. |   by Rewrite11 by a = m ∈ {u:ℤ| P(u) }
. |
. .\
. . 14. ¬a = m
. . ⊢  ∃a:{u:ℤ| P(u) & ¬u = m }. (Replace value k by f(m) in f)(a) = b
. . | by Analyze10 THEN Witness: a
. . |
. . 10. a : ℤ
. . 11. P(a)
. . 12. f(a) = b ∈ {v:ℤ| Q(v) }
. . 13. c : {u:ℤ| P(u) }
. . 14. f(c) = k
. . 15. ¬a = m
. . ⊢  (Replace value k by f(m) in f)(a) = b
. . | by Rewrite by Thm: delete_fenum_value_comp2_gen
. . |
. . .....rewrite subgoal .....
. . ⊢  ¬f(a) = k  by Analyze9
END
```