```
dep_finite_choice  THM

⊢  ∀k:ℕ, B:(ℕk→Type), Q:(x:ℕk→B(x)→Prop).
|  (∀x:ℕk. ∃y:B(x). Q(x;y)) ⇒ (∃f:(x:ℕk→B(x)). ∀x:ℕk. Q(x;f(x)))
| by CompNatInd Concl
|
1. k : ℕ
2. ∀k1:ℕ.
|  k1<k
|  ⇒
|  (∀B:(ℕk1→Type), Q:(x:ℕk1→B(x)→Prop).
|   (∀x:ℕk1. ∃y:B(x). Q(x;y))
|   ⇒
|   (∃f:(x:ℕk1→B(x)). ∀x:ℕk1. Q(x;f(x))))
3. B : ℕk→Type
4. Q : x:ℕk→B(x)→Prop
5. ∀x:ℕk. ∃y:B(x). Q(x;y)
⊢  ∃f:(x:ℕk→B(x)). ∀x:ℕk. Q(x;f(x))  by Decide: k = 0
|
|\
| 6. k = 0
| ⊢  ∃f:(x:ℕk→B(x)). ∀x:ℕk. Q(x;f(x))
|   by {Trivial since there is no ℕ0 } Witness: λx.whatever
|
.\
. 6. ¬k = 0
. ⊢  ∃f:(x:ℕk→B(x)). ∀x:ℕk. Q(x;f(x))
. | by FwdThru: Hyp:2 on [ ∀x:ℕ(k-1). ∃y:B(x). Q(x;y) ]
. |
. |\
. | 7. x : ℕ(k-1)
. | ⊢  ∃y:B(x). Q(x;y)  by BackThru: Hyp:5
. |
. .\
. . 7. ∃f:(x:ℕ(k-1)→B(x)). ∀x:ℕ(k-1). Q(x;f(x))
. . ⊢  ∃f:(x:ℕk→B(x)). ∀x:ℕk. Q(x;f(x))
. . | by {get a function assumed to exist for x ∈ ℕ(k-1) }
. . |    New:g Analyze-1
. . |
. . 7. g : x:ℕ(k-1)→B(x)
. . 8. ∀x:ℕ(k-1). Q(x;g(x))
. . ⊢  ∃f:(x:ℕk→B(x)). ∀x:ℕk. Q(x;f(x))
. . | by {get a witness for x = k-1 ∈ ℕk }
. . |    Inst: Hyp:5 Using:[k-1] THEN Analyze-1
. . |
. . 9. y : B(k-1)
. . 10. Q(k-1;y)
. . ⊢  ∃f:(x:ℕk→B(x)). ∀x:ℕk. Q(x;f(x))
. . | by SimilarTo:
. . |    ∃f:(x:ℕk→B(x)). f(k-1) = y & (∀x:ℕ(k-1). f(x) = g(x))
. . |
. . |\
. . | .....assertion .....
. . | ⊢  ∃f:(x:ℕk→B(x)). f(k-1) = y & (∀x:ℕ(k-1). f(x) = g(x))
. . | | by Witness: λx.if x=₂k-1→ y else g(x) fi with type x:ℕk→B(x)
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. . | |    THEN
. . | |    Reduce Concl
. . | |
. . | |\
. . | | .....assertion .....
. . | | ⊢  (λx.if x=₂k-1→ y else g(x) fi) ∈ x:ℕk→B(x)
. . | |   by SideProof 1
. . | |
. . | .\
. . | . 11. (λx.if x=₂k-1→ y else g(x) fi) ∈ x:ℕk→B(x)
. . | . ⊢  if k-1=₂k-1→ y else g(k-1) fi = y
. . | .    & (∀x:ℕ(k-1). if x=₂k-1→ y else g(x) fi = g(x))
. . | .   by Repeat (SplitITE Concl)
. . |
. . .\
. . . 11. f : x:ℕk→B(x)
. . . 12. f(k-1) = y
. . . 13. ∀x:ℕ(k-1). f(x) = g(x)
. . . 14. x : ℕk
. . . ⊢  Q(x;f(x))  by Decide: x = k-1 ∈ ℕk
. . . |
. . . |\
. . . | 15. x = k-1 ∈ ℕk
. . . | ⊢  Q(x;f(x))  by Rewrite by Q(x;f(x)) = Q(k-1;y) ∈ Prop
. . . | |
. . . | .....assertion .....
. . . | ⊢  Q(x;f(x)) = Q(k-1;y) ∈ Prop  by Repeat Analyze
. . . | |
. . . | ⊢  f(x) = y ∈ B(x)  by Hyp:12  Asserted
. . . | |
. . . | 16. f(k-1) = y
. . . | ⊢  f(x) = y ∈ B(x)  by ChangeToEqType-1: B(x)
. . . | |
. . . | |\
. . . | | 16. f(k-1) = y ∈ B(x)
. . . | | ⊢  f(x) = y ∈ B(x)  by ApFun: f to: x = k-1 ∈ ℕk
. . . | |
. . . | .\
. . . | . .....equality .....
. . . | . ⊢  B(k-1) = B(x) ∈ Type  by Analyze
. . . |
. . . .\
. . . . 15. ¬x = k-1 ∈ ℕk
. . . . ⊢  Q(x;f(x))  by Inst: Hyp:13 Using:[x]
. . . . |
. . . . 16. f(x) = g(x)
. . . . ⊢  Q(x;f(x))  by Rewrite by Hyp:-1

. . . . |
. . . . ⊢  Q(x;g(x))  by BackThru: Hyp:8
END
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dep_finite_choice Side Proof 1  THM

.....assertion .....
1. k : ℕ
2. ∀k1:ℕ.
|  k1<k
|  ⇒
|  (∀B:(ℕk1→Type), Q:(x:ℕk1→B(x)→Prop).
|   (∀x:ℕk1. ∃y:B(x). Q(x;y))
|   ⇒
|   (∃f:(x:ℕk1→B(x)). ∀x:ℕk1. Q(x;f(x))))
3. B : ℕk→Type
4. Q : x:ℕk→B(x)→Prop
5. ∀x:ℕk. ∃y:B(x). Q(x;y)
6. ¬k = 0
7. g : x:ℕ(k-1)→B(x)
8. ∀x:ℕ(k-1). Q(x;g(x))
9. y : B(k-1)
10. Q(k-1;y)
⊢  (λx.if x=₂k-1→ y else g(x) fi) ∈ x:ℕk→B(x)  by Analyze
|
11. x : ℕk
⊢  if x=₂k-1→ y else g(x) fi ∈ B(x)  by SplitITE Concl
|
.....truecase .....
12. x = k-1
⊢  y ∈ B(x)  by ChangeToEqType: B(k-1)
|
.....equality .....
⊢  B(x) = B(k-1) ∈ Type  by Analyze
END
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