```
finite_choice  THM

⊢  ∀A:Type, k:ℕ, Q:(ℕk→A→Prop).
|  (∀x:ℕk. ∃y:A. Q(x;y)) ⇒ (∃f:(ℕk→A). ∀x:ℕk. Q(x;f(x)))
| by Analyze THEN CompNatInd Concl
|
1. A : Type
2. k : ℕ
3. ∀k1:ℕ.
|  k1<k
|  ⇒
|  (∀Q:(ℕk1→A→Prop).
|   (∀x:ℕk1. ∃y:A. Q(x;y)) ⇒ (∃f:(ℕk1→A). ∀x:ℕk1. Q(x;f(x))))
4. Q : ℕk→A→Prop
5. ∀x:ℕk. ∃y:A. Q(x;y)
⊢  ∃f:(ℕk→A). ∀x:ℕk. Q(x;f(x))  by Decide: k = 0 ∈ ℕ
|
|\
| 6. k = 0 ∈ ℕ
| ⊢  ∃f:(ℕk→A). ∀x:ℕk. Q(x;f(x))  by Witness: λx.whatever
|
.\
. 6. ¬k = 0 ∈ ℕ
. ⊢  ∃f:(ℕk→A). ∀x:ℕk. Q(x;f(x))  by Inst: Hyp:3 Using:[k-1 | Q]
. |
. |\
. | 7. x : ℕ(k-1)
. | ⊢  ∃y:A. Q(x;y)  by Witness5: x
. |
. .\
. . 7. ∃f:(ℕ(k-1)→A). ∀x:ℕ(k-1). Q(x;f(x))
. . ⊢  ∃f:(ℕk→A). ∀x:ℕk. Q(x;f(x))  by New:g Analyze-1
. . |
. . 7. g : ℕ(k-1)→A
. . 8. ∀x:ℕ(k-1). Q(x;g(x))
. . ⊢  ∃f:(ℕk→A). ∀x:ℕk. Q(x;f(x))
. . | by (SimilarTo:
. . |     ∃f:(ℕk→A). f(k-1) = y & (∀x:ℕ(k-1). f(x) = g(x)) ;
. . |     Witness5: k-1)
. . |    THEN
. . |    Analyze-1
. . |
. . 9. y : A
. . 10. Q(k-1;y)
. . ⊢  ∃f:(ℕk→A). ∀x:ℕk. Q(x;f(x))
. . | by SimilarTo: ∃f:(ℕk→A). f(k-1) = y & (∀x:ℕ(k-1). f(x) = g(x))
. . |
. . |\
. . | .....assertion .....
. . | ⊢  ∃f:(ℕk→A). f(k-1) = y & (∀x:ℕ(k-1). f(x) = g(x))
. . | | by Witness: λx.if x=₂k-1→ y else g(x) fi with type ℕk→A
. . | |    THENP
. . | |    (Analyze THEN SplitITE Concl)
. . | |
. . | 11. (λx.if x=₂k-1→ y else g(x) fi) ∈ ℕk→A
```

```
. . | ⊢  if k-1=₂k-1→ y else g(k-1) fi = y
. . |    & (∀x:ℕ(k-1). if x=₂k-1→ y else g(x) fi = g(x))
. . |   by Repeat SplitOnConclITE
. . |
. . .\
. . . 11. f : ℕk→A
. . . 12. f(k-1) = y
. . . 13. ∀x:ℕ(k-1). f(x) = g(x)
. . . 14. x : ℕk
. . . ⊢  Q(x;f(x))  by Decide: x = k-1 ∈ ℕk
. . . |
. . . |\
. . . | 15. x = k-1 ∈ ℕk
. . . | ⊢  Q(x;f(x))
. . . |   by Rewrite by Q(x;f(x)) = Q(k-1;y) ∈ Prop
. . . |      THEN
. . . |      Repeat Analyze
. . . |      THEN
. . . |      ApFun: f to: x = k-1 ∈ ℕk
. . . |
. . . .\
. . . . 15. ¬x = k-1 ∈ ℕk
. . . . ⊢  Q(x;f(x))  by Inst: Hyp:13 Using:[x]
. . . . |
. . . . 16. f(x) = g(x)
. . . . ⊢  Q(x;f(x))  by Rewrite by Hyp:-1
. . . . |
. . . . ⊢  Q(x;g(x))  by BackThru: Hyp:8
END
```