```
finite_inj_counter_example  THM

⊢  ∀m:ℕ, f:(ℕm→ℤ). ¬Inj(ℕm; ℤ; f) ⇒ (∃x:ℕm, y:ℕx. f(x) = f(y))
| by Auto
|
1. m : ℕ
2. f : ℕm→ℤ
3. ¬Inj(ℕm; ℤ; f)
⊢  ∃x:ℕm, y:ℕx. f(x) = f(y)
| by Decide: ∃x:ℕm, y:ℕx. f(x) = f(y) ∈ ℤ THEN Analyze3
|
4. ¬(∃x:ℕm, y:ℕx. f(x) = f(y))
⊢  Inj(ℕm; ℤ; f)  by Analyze
|
5. a1 : ℕm
6. a2 : ℕm
7. f(a1) = f(a2)
⊢  a1 = a2  by ¬a1<a2 & ¬a2<a1  Asserted THEN Analyze THEN Analyze4
|
|\
| 8. a1<a2
| ⊢  ∃x:ℕm, y:ℕx. f(x) = f(y)  by Witness: a2 THEN Witness: a1
|
.\
. 8. a2<a1
. ⊢  ∃x:ℕm, y:ℕx. f(x) = f(y)  by Witness: a1 THEN Witness: a2
END
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